frattali

ITADINFO 2023 – materiale didattico

oggetti che ripetono sé stessi

Gaetano Impoco

gaetano @ impoco.it

frattali
obiettivi

comprendere:

1. capacità di un **sistema simbolico** di comunicare info
2. differenza tra **linguaggi naturali** e **linguaggi formali**
3. caratteristiche: **alfabeto**, **sintassi**, **semantica**, grammatica
4. differenza tra **simboli terminali** e **non-terminali**
5. **autoreferenzialità**: parole che si riferiscono a sé stesse

imparare:

6. **rappresentazioni** grafiche e simboliche
7. “**modellare**” casi reali

recap

linguaggi formali
vecchie conoscenze…

ingredienti

1. alfabeto $\Sigma = \{ \mathbf{a}, \mathbf{s}, \mathbf{d} \}_{90°}$

2. interpretazione

   a: avanti

   s: sinistra

   d: destra

3. parola **A → asadadasa**

recap

linguaggi formali
vecchie conoscenze…

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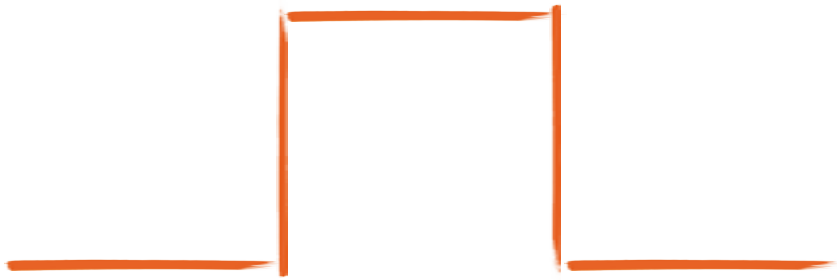
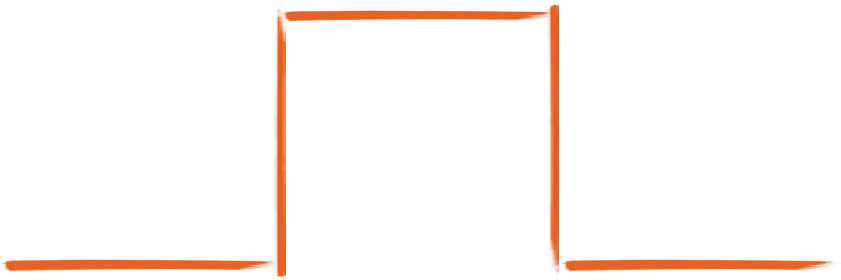


linguaggi formali
stranezze



$$\Sigma = \{ a, s, d \}_{90°}$$

A → asadadasa

A → AsAdAdAsA

linguaggi formali
espansione

che succede se uso questa regola? **A → AsAdAdAsA**

continuo a “espandere” **A** all'infinito!

soluzione: aggiungo la regola **A → a**

linguaggi formali
espansione

due regole

1. **A → AsAdAdAsA**
2. **A → a**

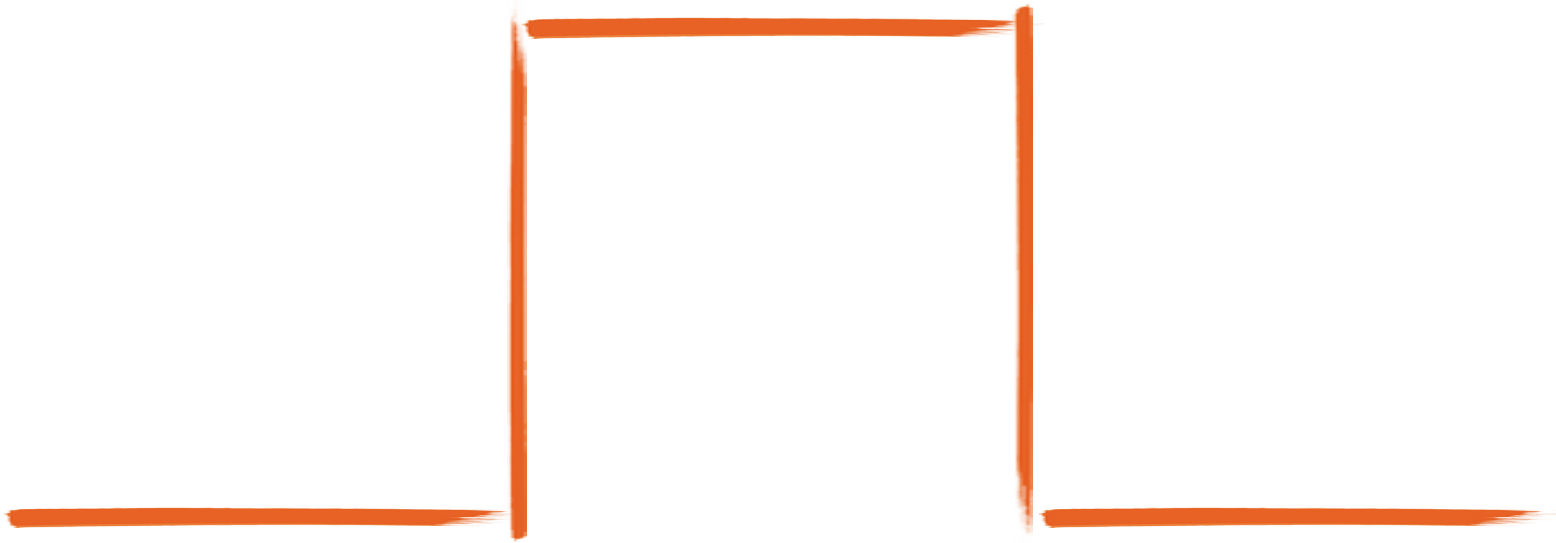
simbolo iniziale: **A**

linguaggi formali
ricorsione

disegnamo!

S → **(1)** **A → AsAdAdAsA**
(2) **A → a**

passo 1 – regole: **1, 2**



linguaggi formali
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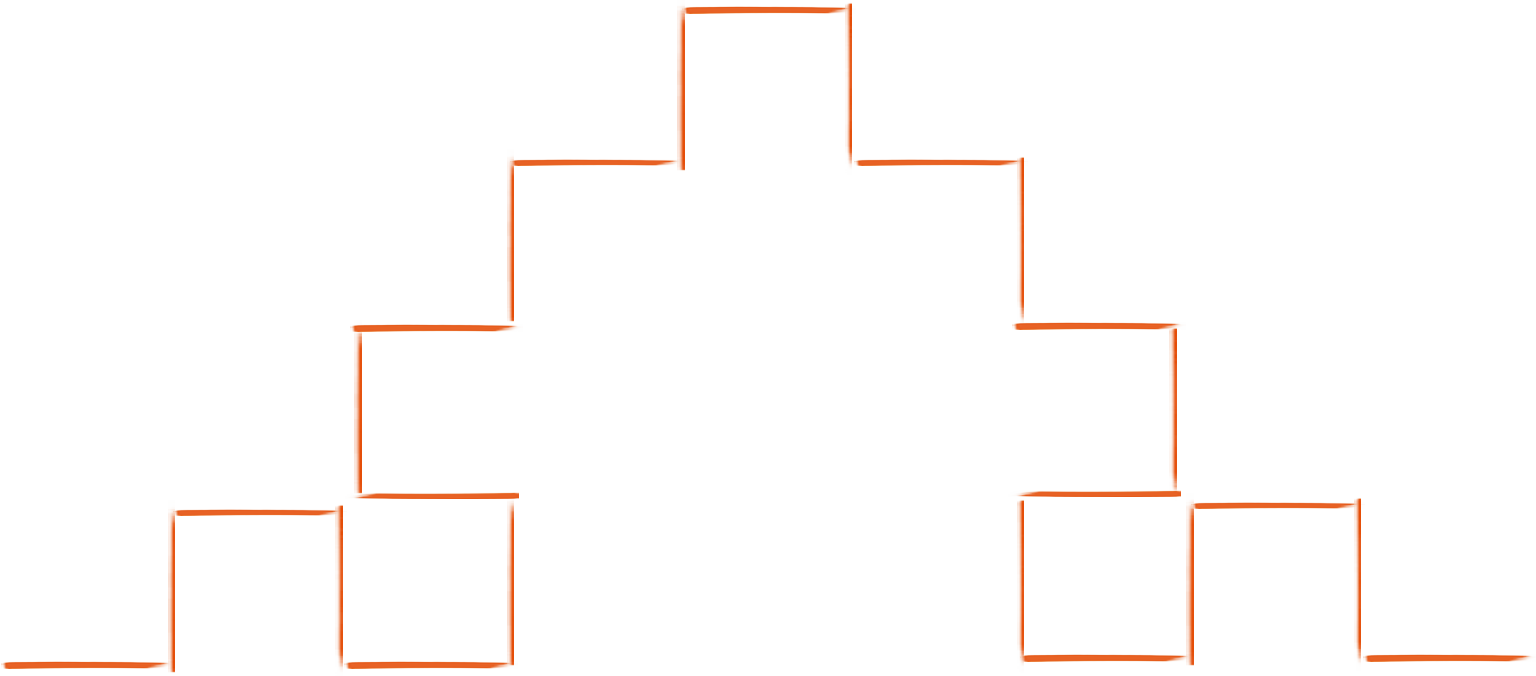

linguaggi formali
ricorsione

disegnamo!

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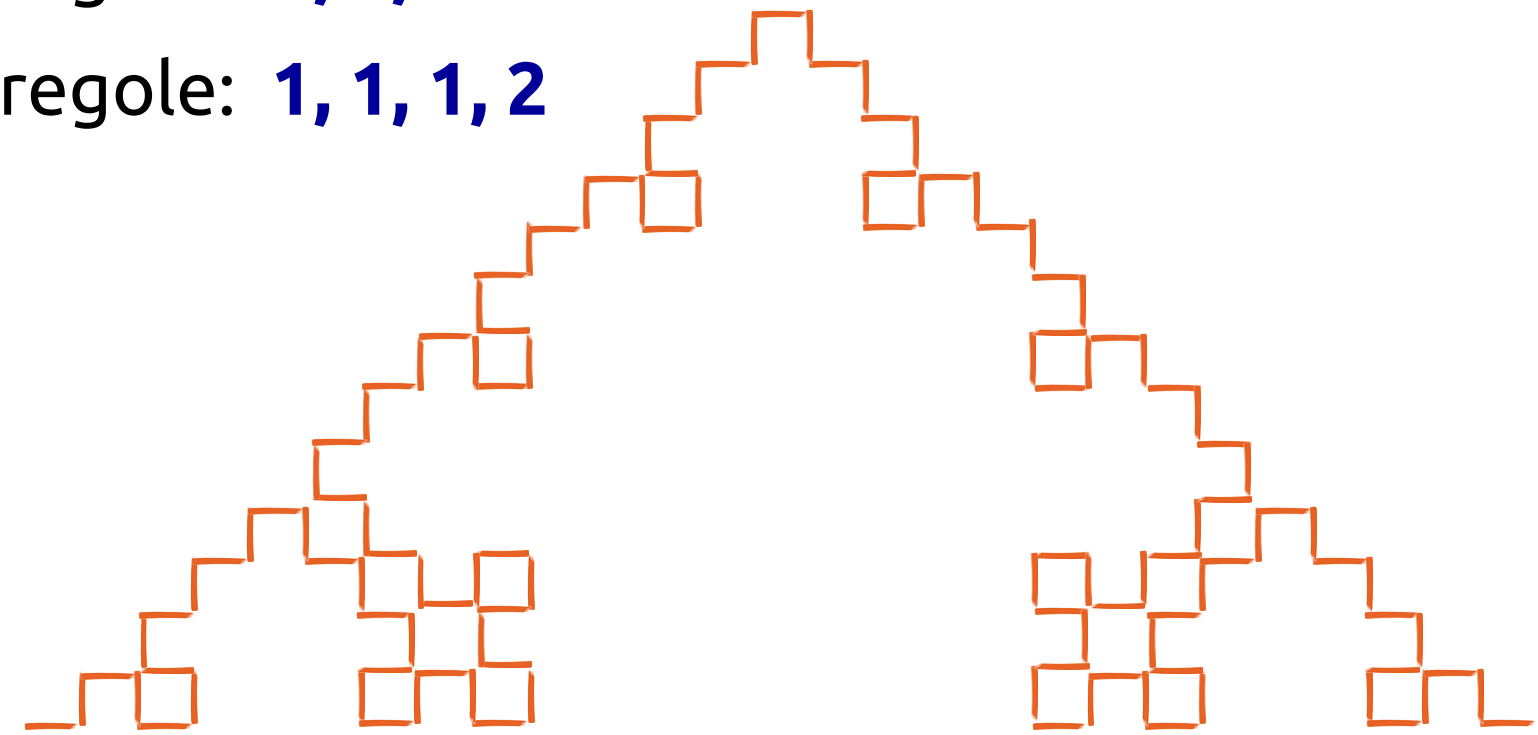
passo 1 – regole: **1, 2**

passo 2 – regole: **1, 1, 2**

passo 3 – regole: **1, 1, 1, 2**

…



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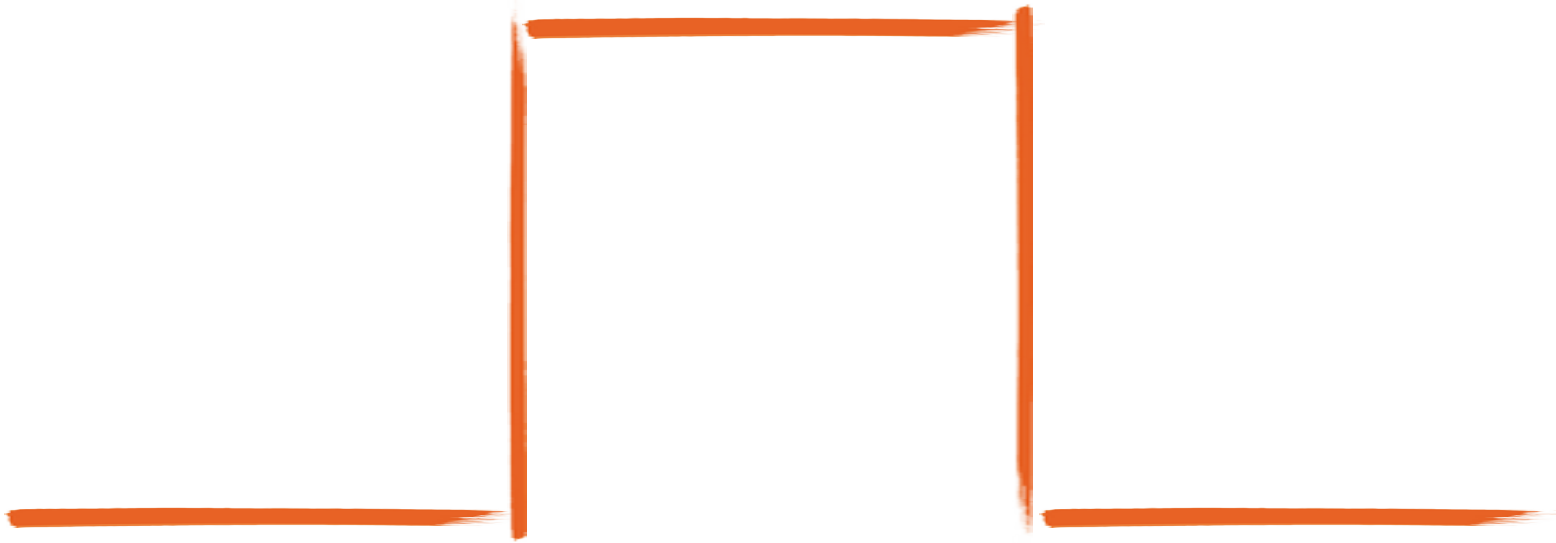

linguaggi formali
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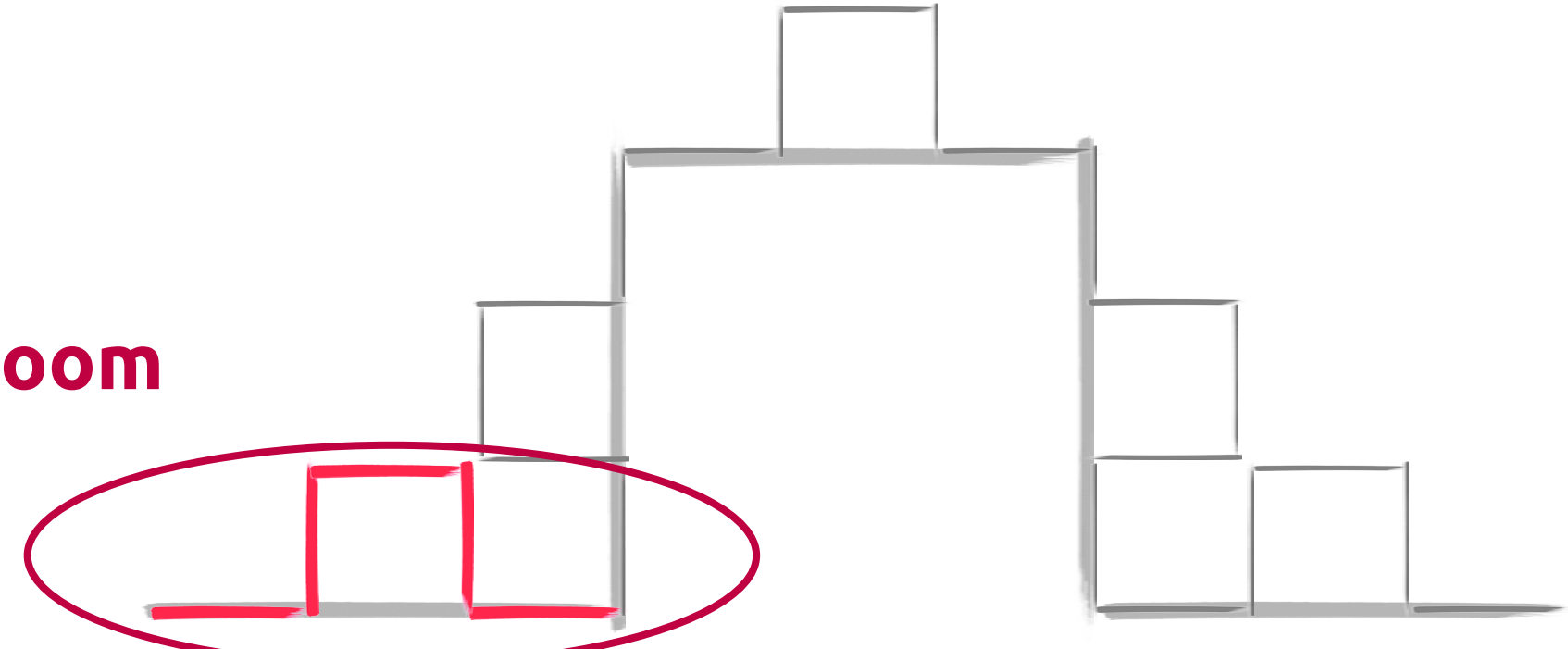
zoom



linguaggi formali
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…

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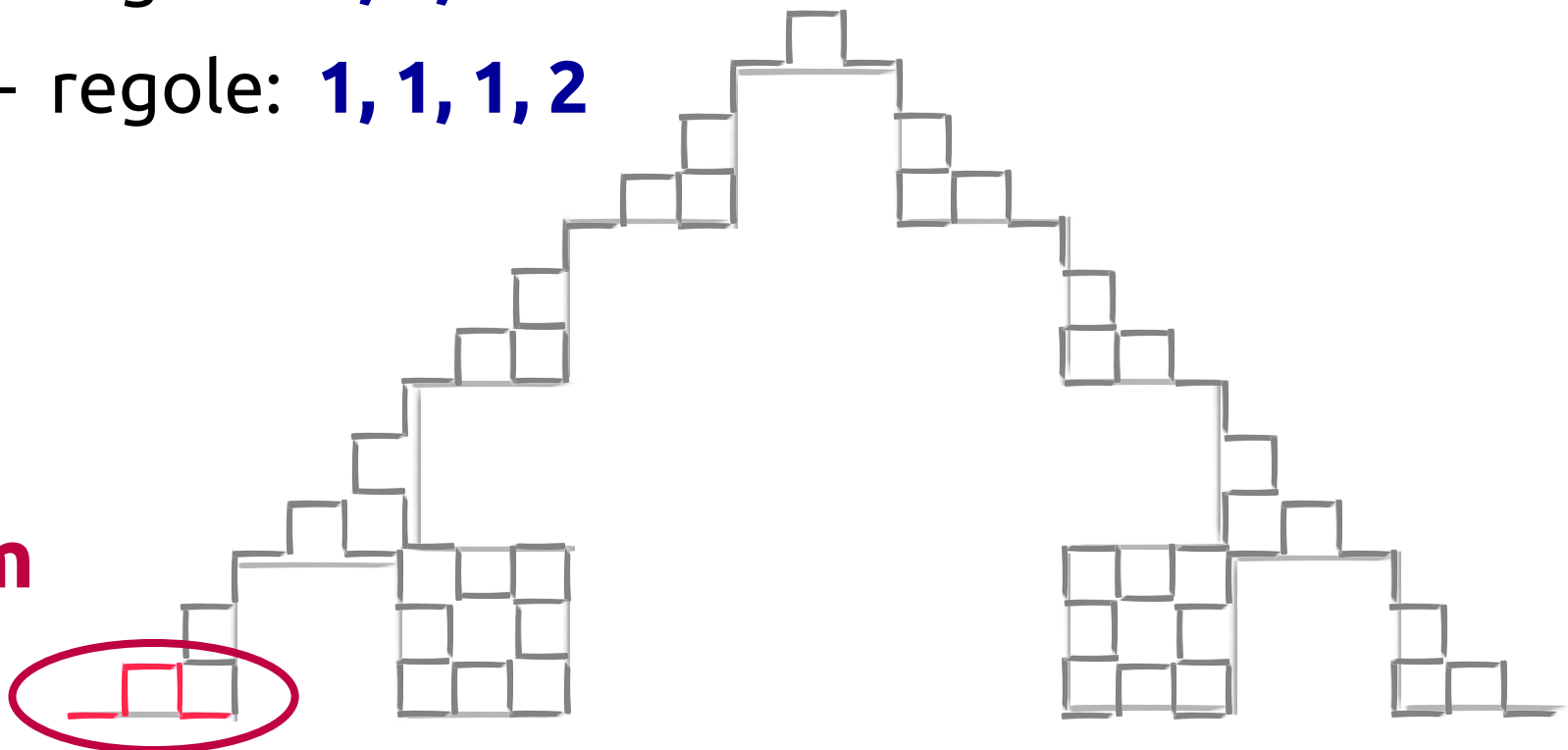

linguaggi formali
ricorsione – curva di Koch

disegnamo!

S → **(1)** **A → AsAddAsA**
(2) **A → a**

passo 1 – regole: **1, 2**

linguaggi formali
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passo 1 – regole: **1, 2**

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linguaggi formali
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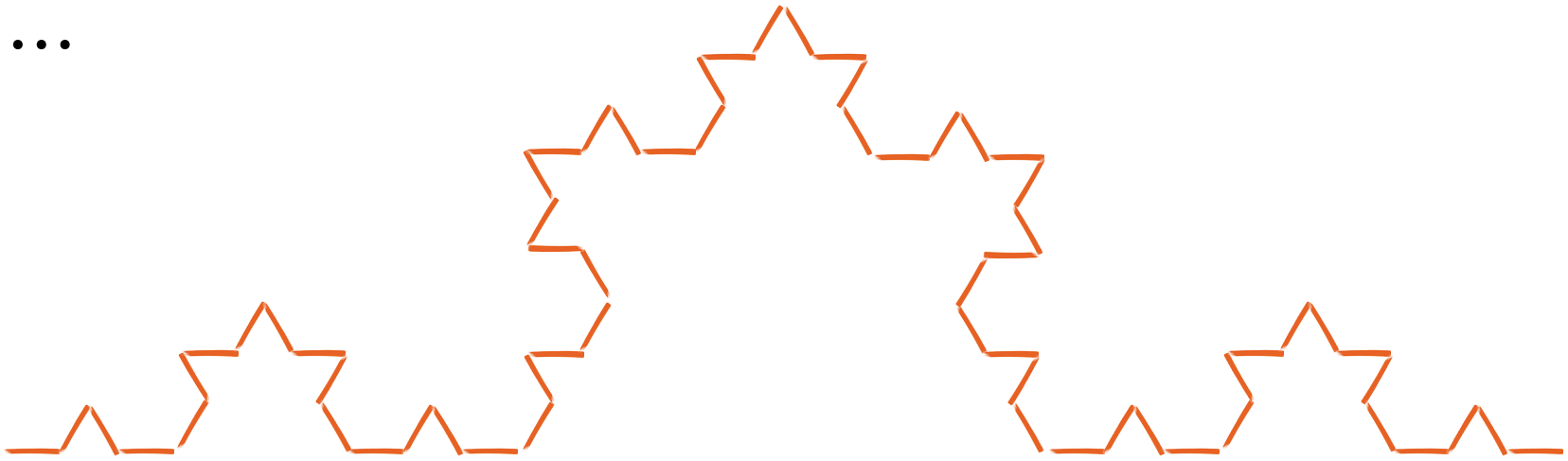

frattali

frattali
cosa sono?

figure geometriche che si ripetono all'infinito uguale a sé stesse, su scala sempre più piccola

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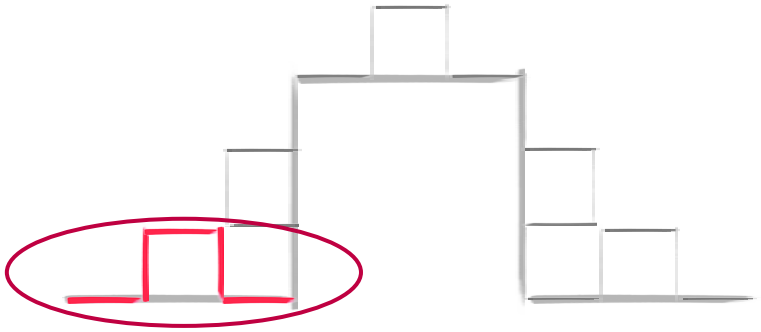
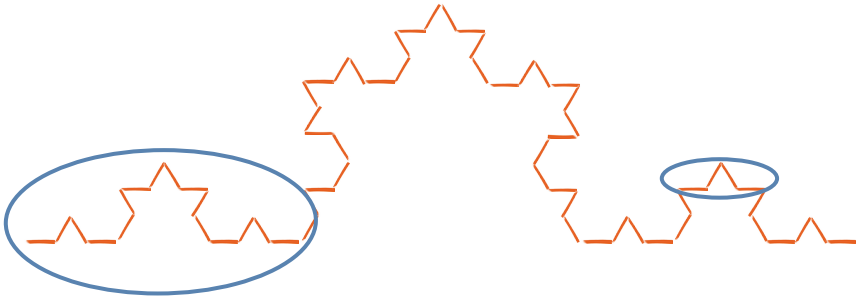
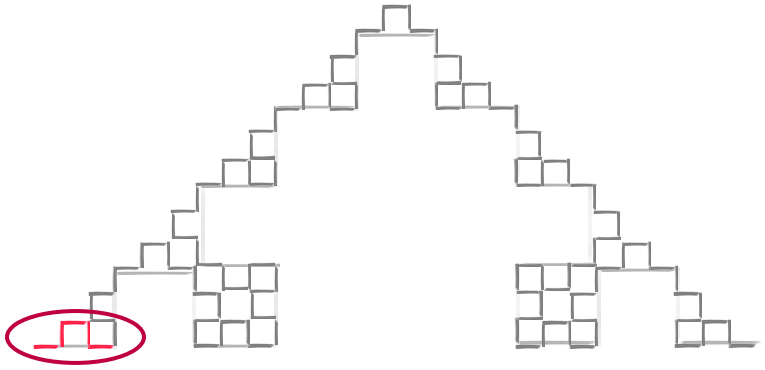
figure geometriche che si ripetono all'infinito uguale a sé stesse, su scala sempre più piccola







vecchie conoscenze!

frattali
esercizio #9

1. dividetevi in coppie
2. cercate sul web esempi di frattali in natura
3. condividete postando sulla pagina:

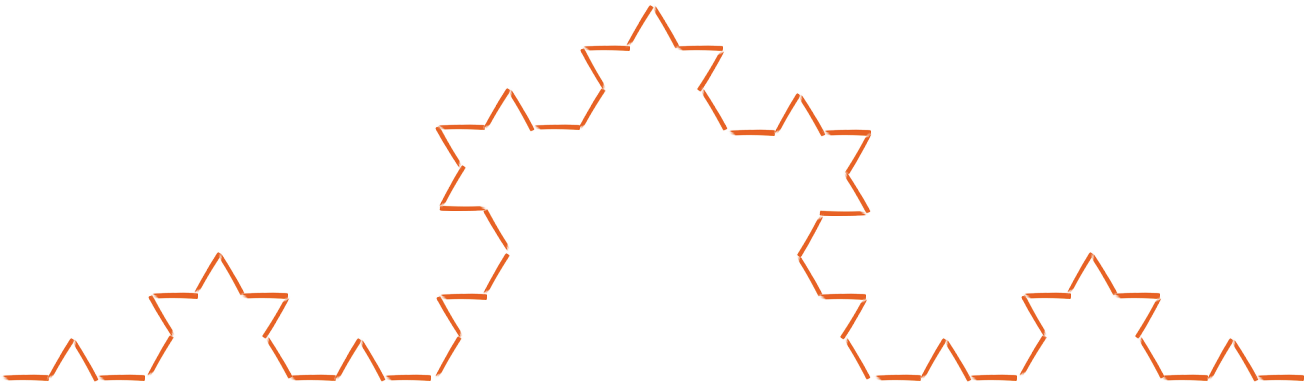
    url di uno strumento di condivisione (padlet, lavagna, …)

frattali
… e linguaggi formali

- grammatica: alfabeto + regole
- regole “ricorsive”
  - **A → A s A d d A s A**

autosimilarità

“la parte è copia del tutto”



frattali
proprietà

1. autosimilarità

   il frattale è unione di copie di sé stesso a scale differenti

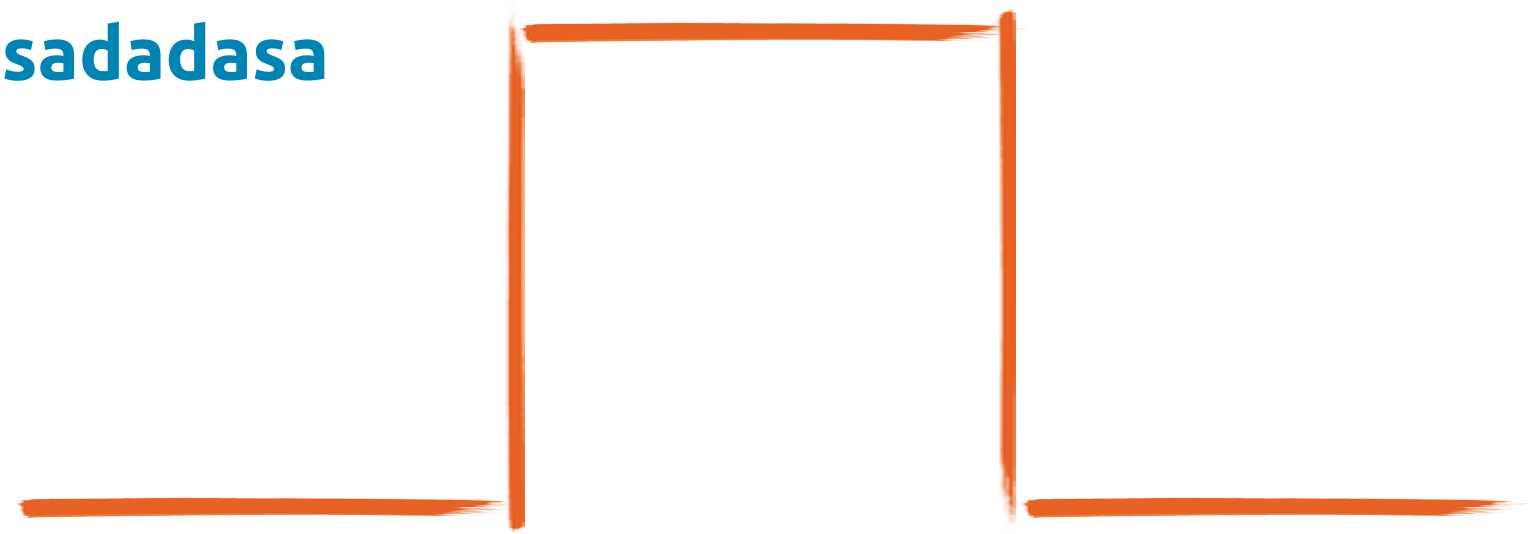
2. struttura fine

   rivela dettagli ad ogni ingrandimento

3. … sorpresa!

frattali
quanta strada percorre la tartaruga?

A → asadadasa



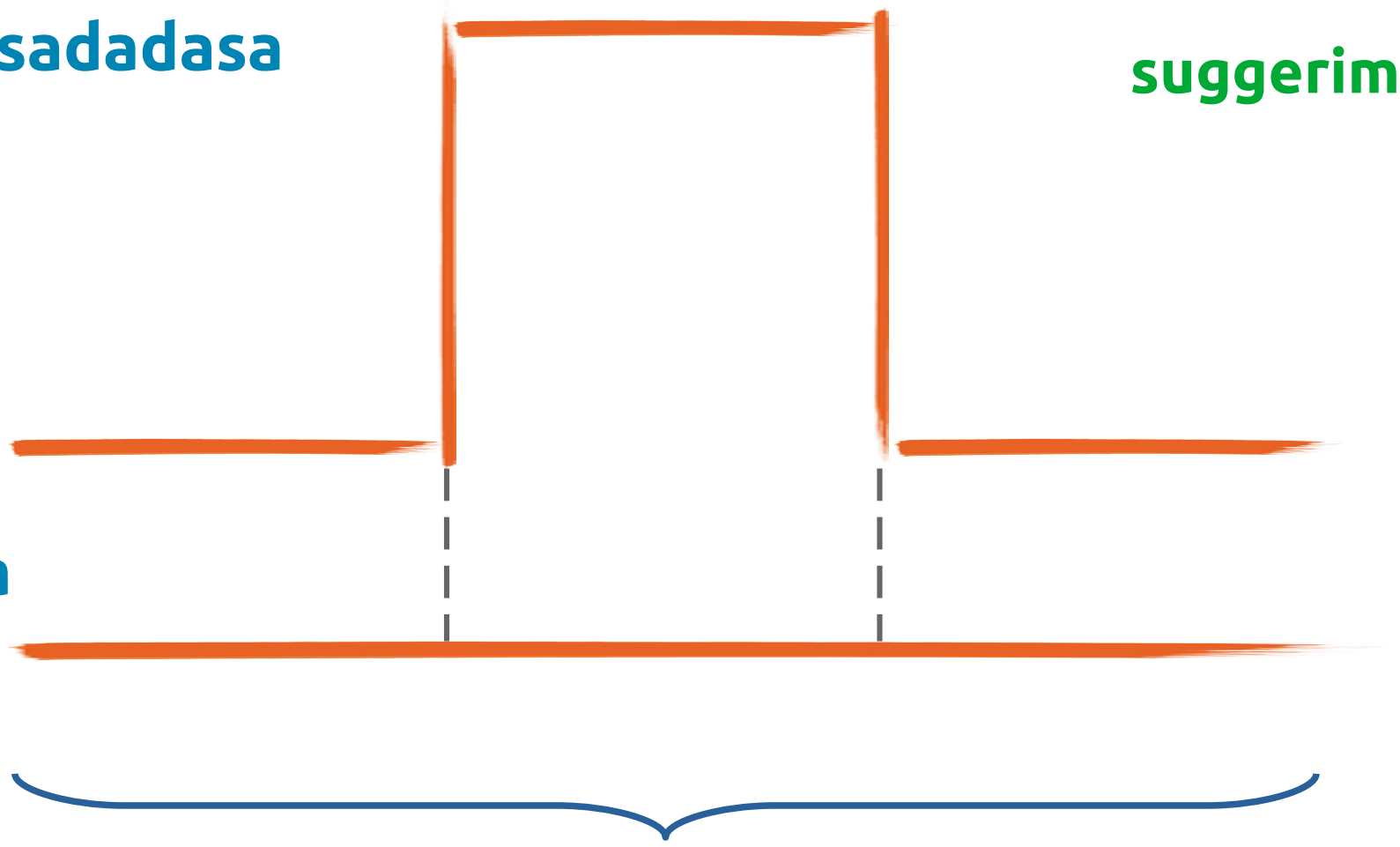
A → a

lunghezza = 1

frattali
quanta strada percorre la tartaruga?

A → asadadasa

suggerimento

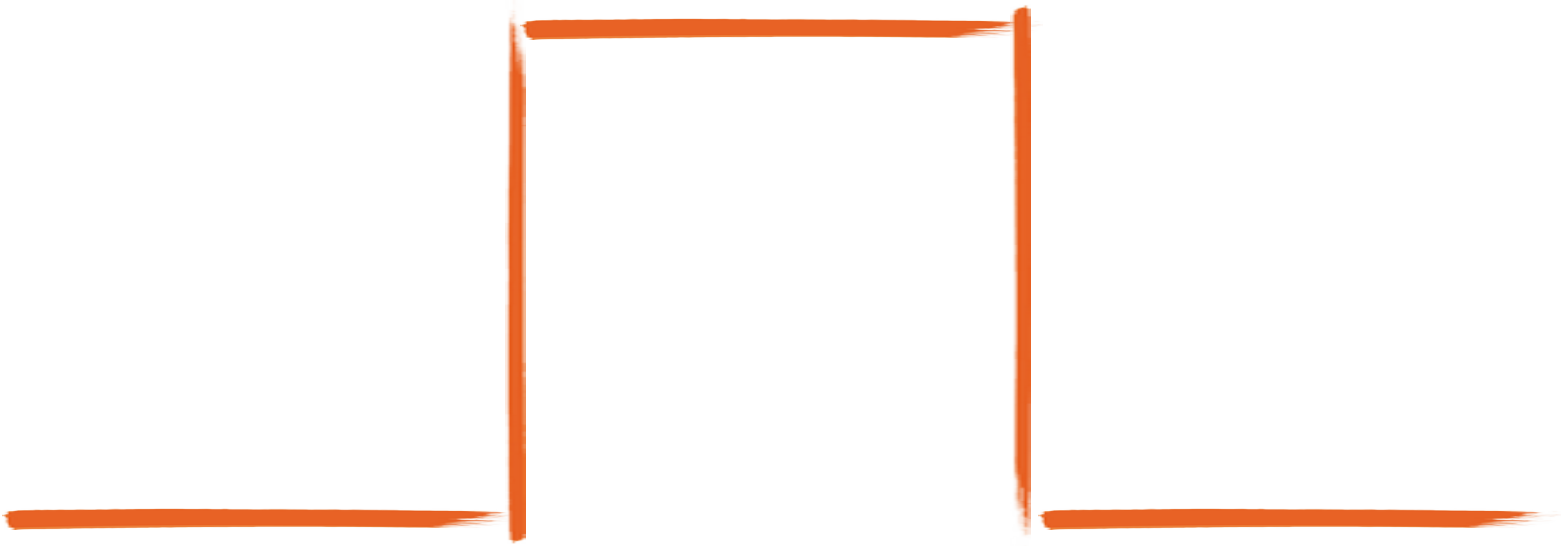


A → a

lunghezza = 1

frattali
esercizio #10

1. dividetevi in coppie
2. provate a calcolare la lunghezza della curva



frattali
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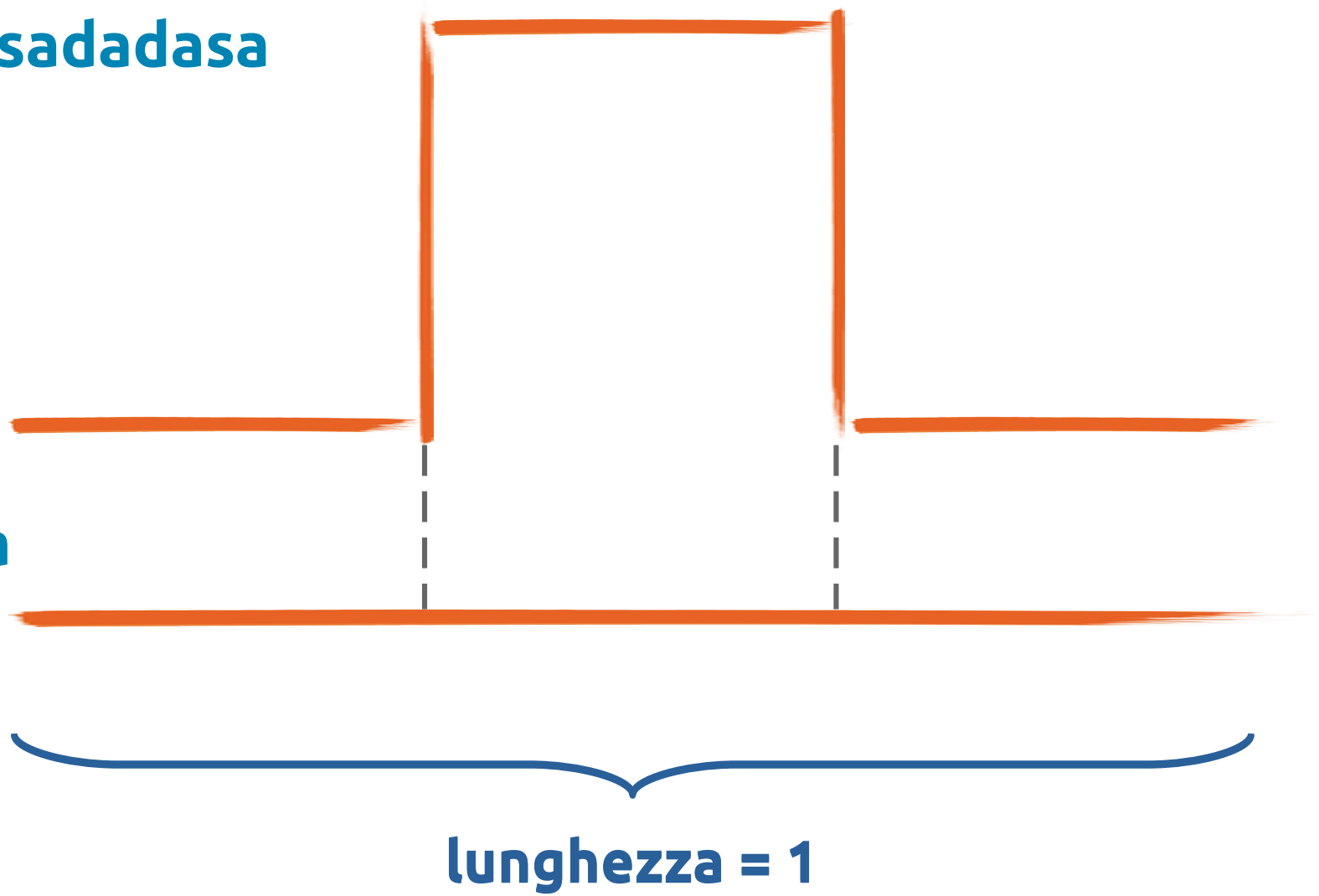

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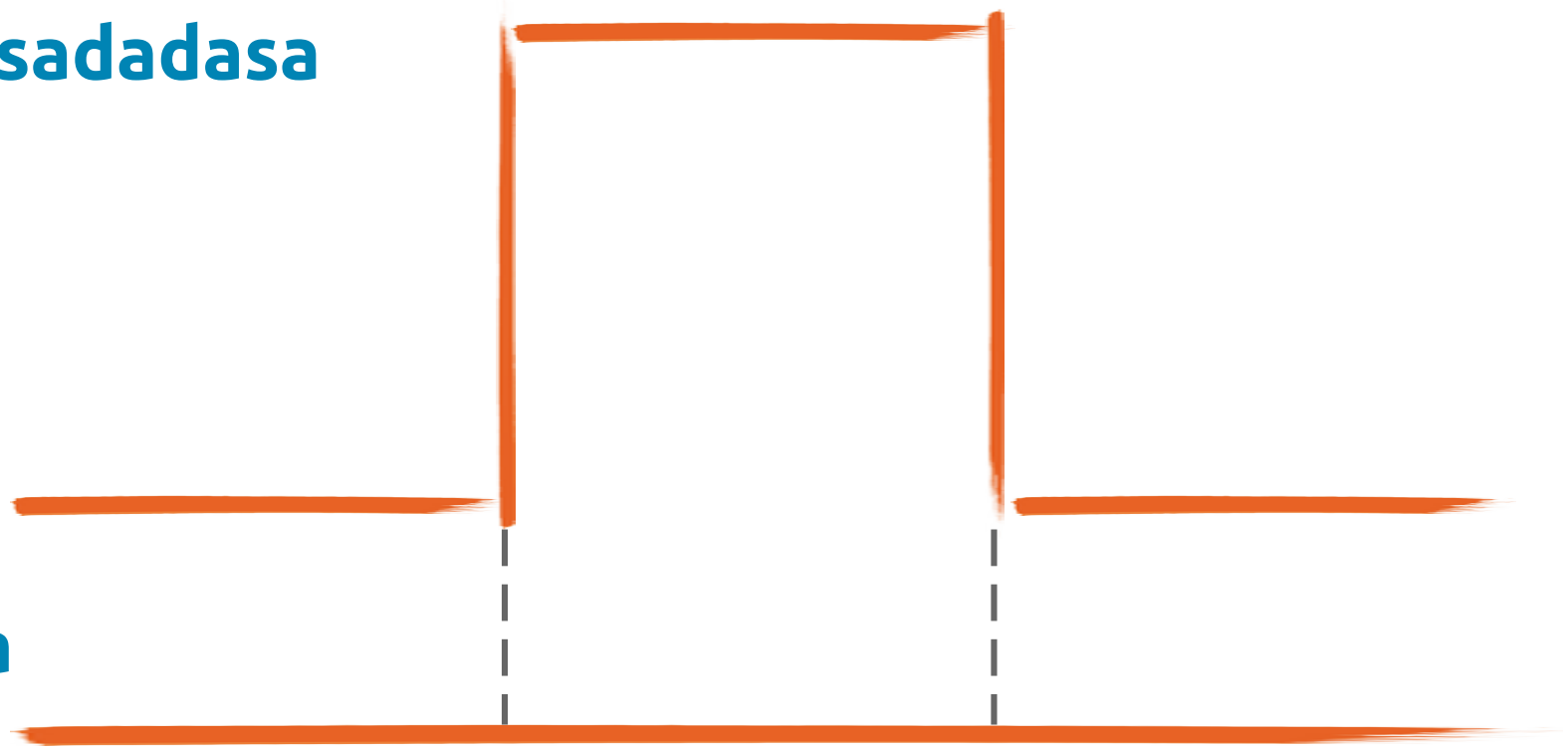

A → a

l = 1/3 **l = 1/3** **l = 1/3**

frattali
quanta strada percorre la tartaruga?

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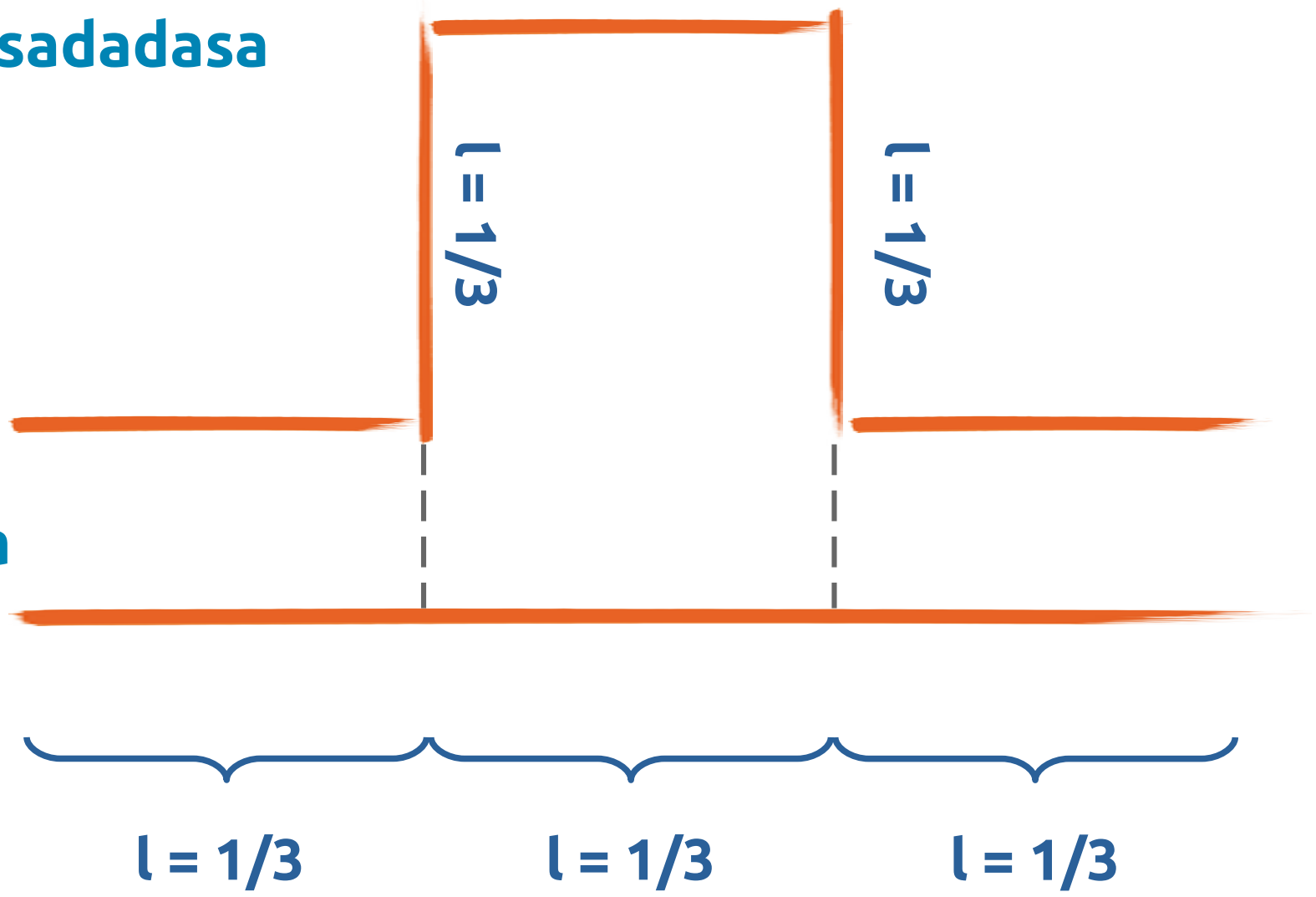

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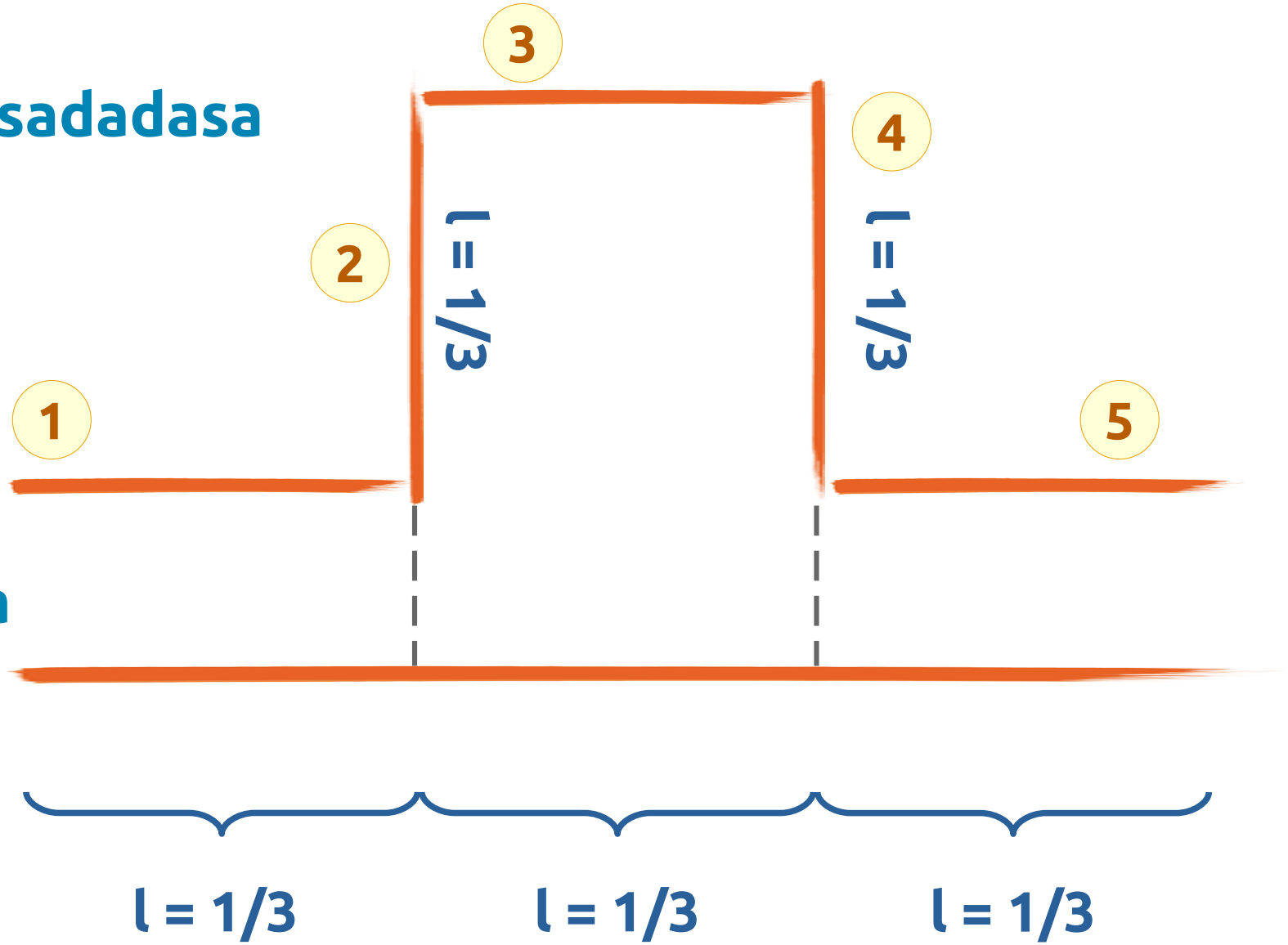
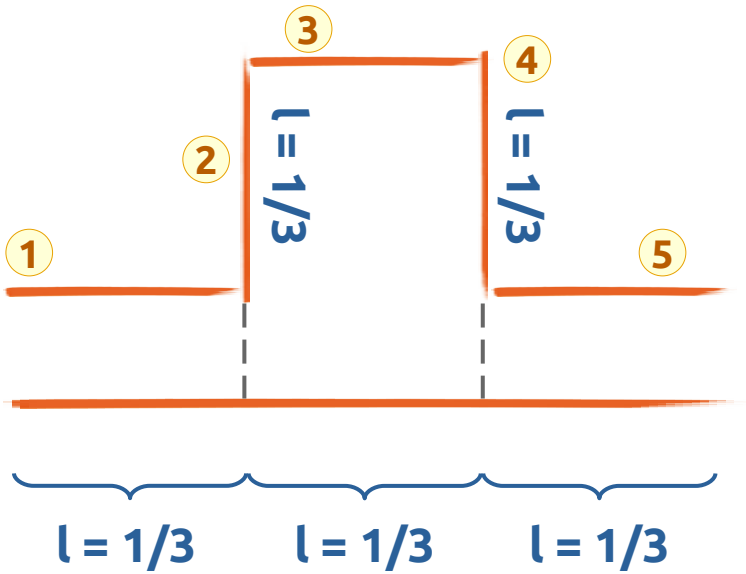


frattali
quanta strada percorre la tartaruga?

5 linee, ciascuna lunga 1/3

→ 5/3



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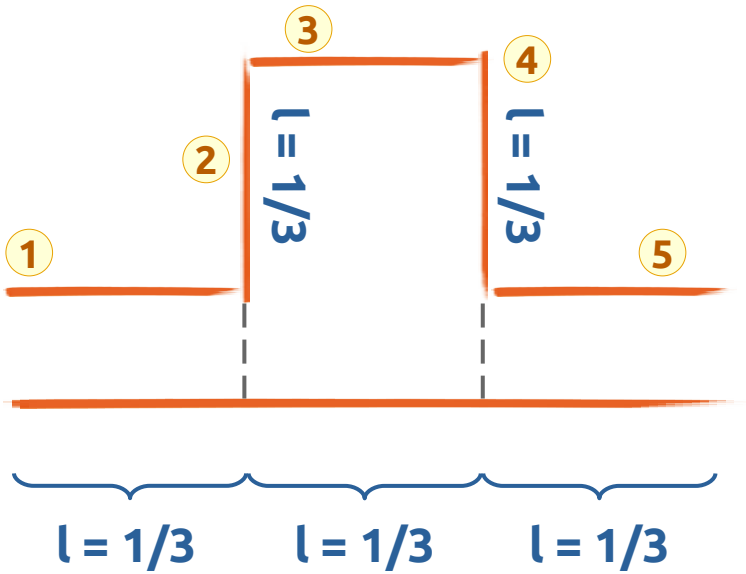

A → asadadasa

1 2 3 4 5

frattali
quanta strada percorre la tartaruga?

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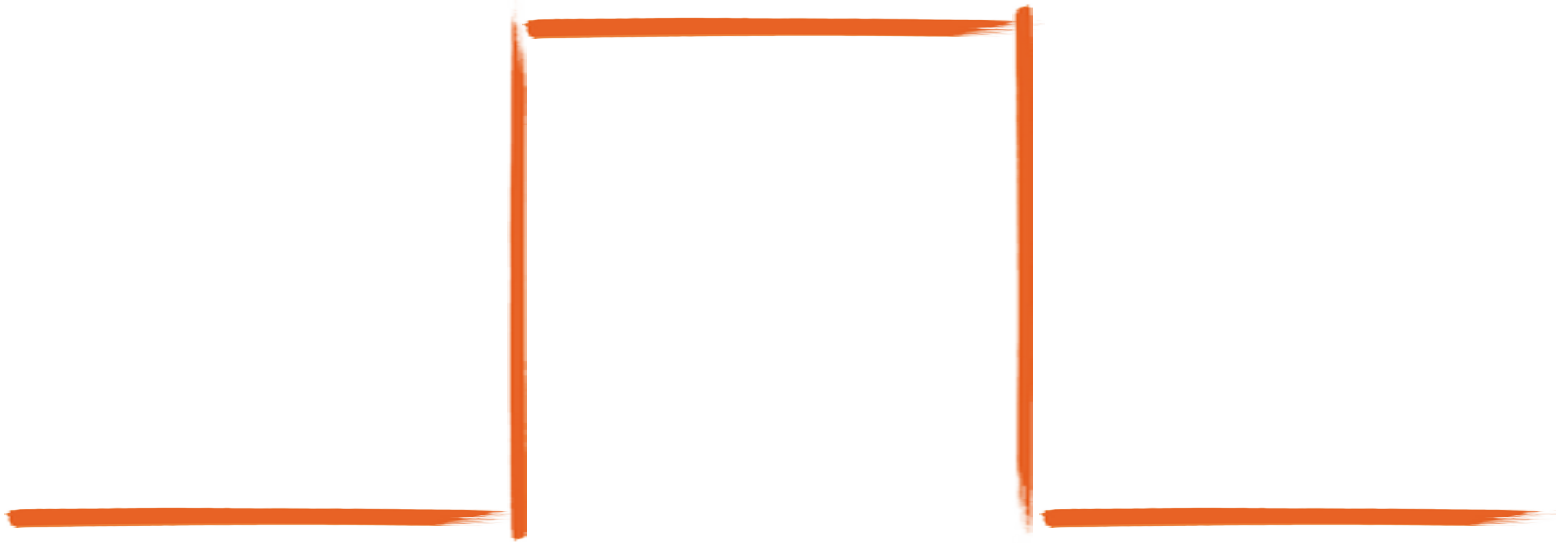
(2) **A → a**

regole: **1, 2**

l = 5/3



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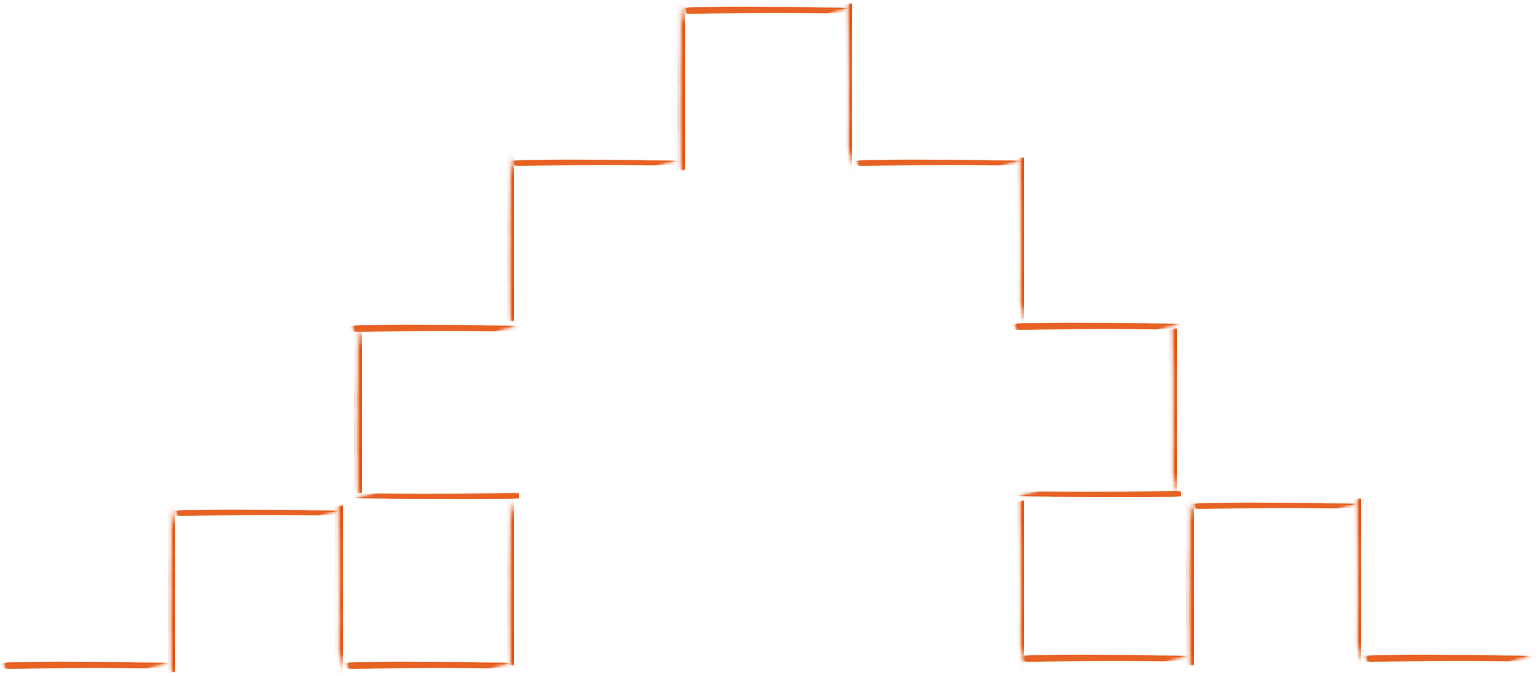

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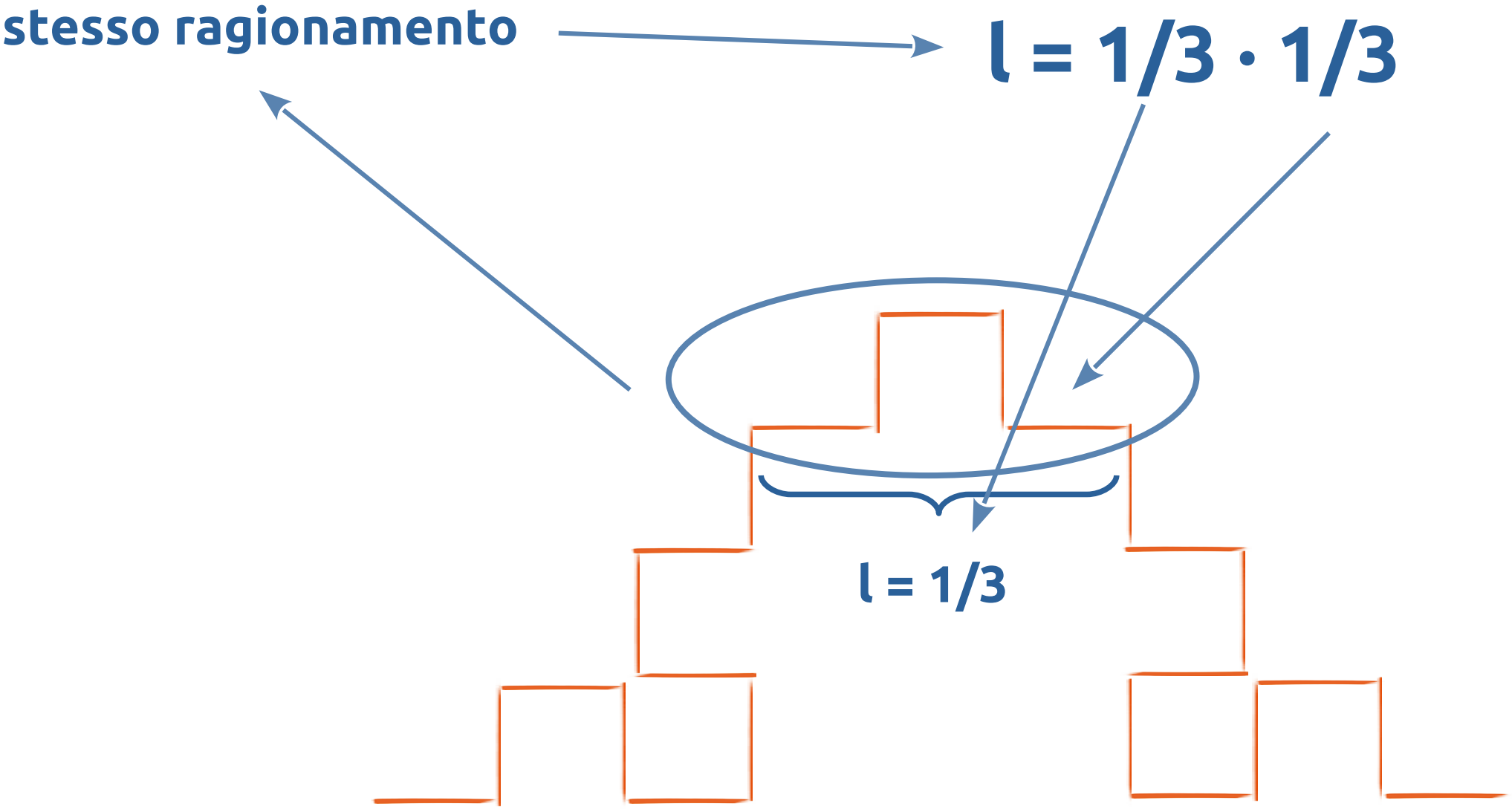
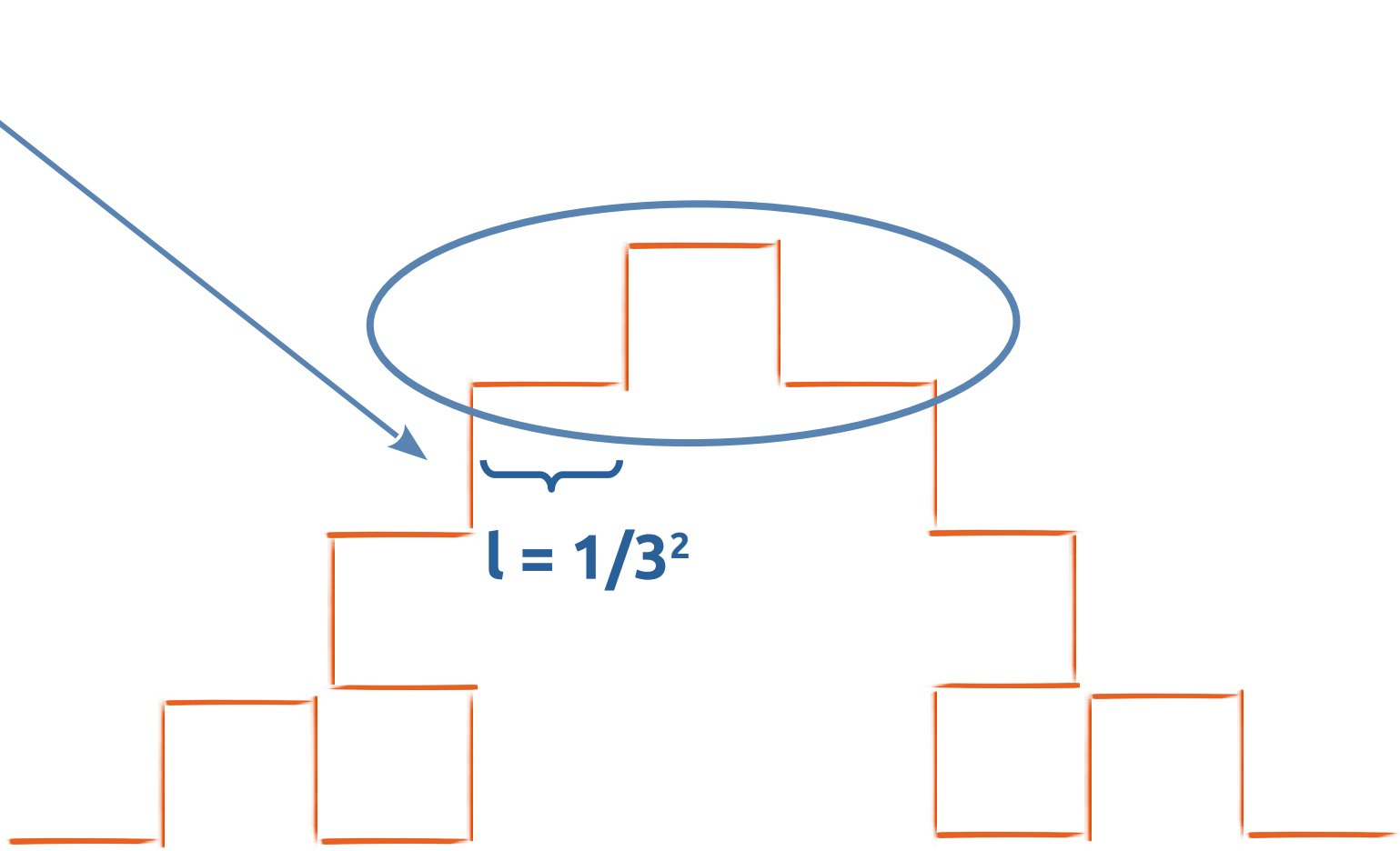


frattali
quanta strada percorre la tartaruga?

$l = 1/3^2$

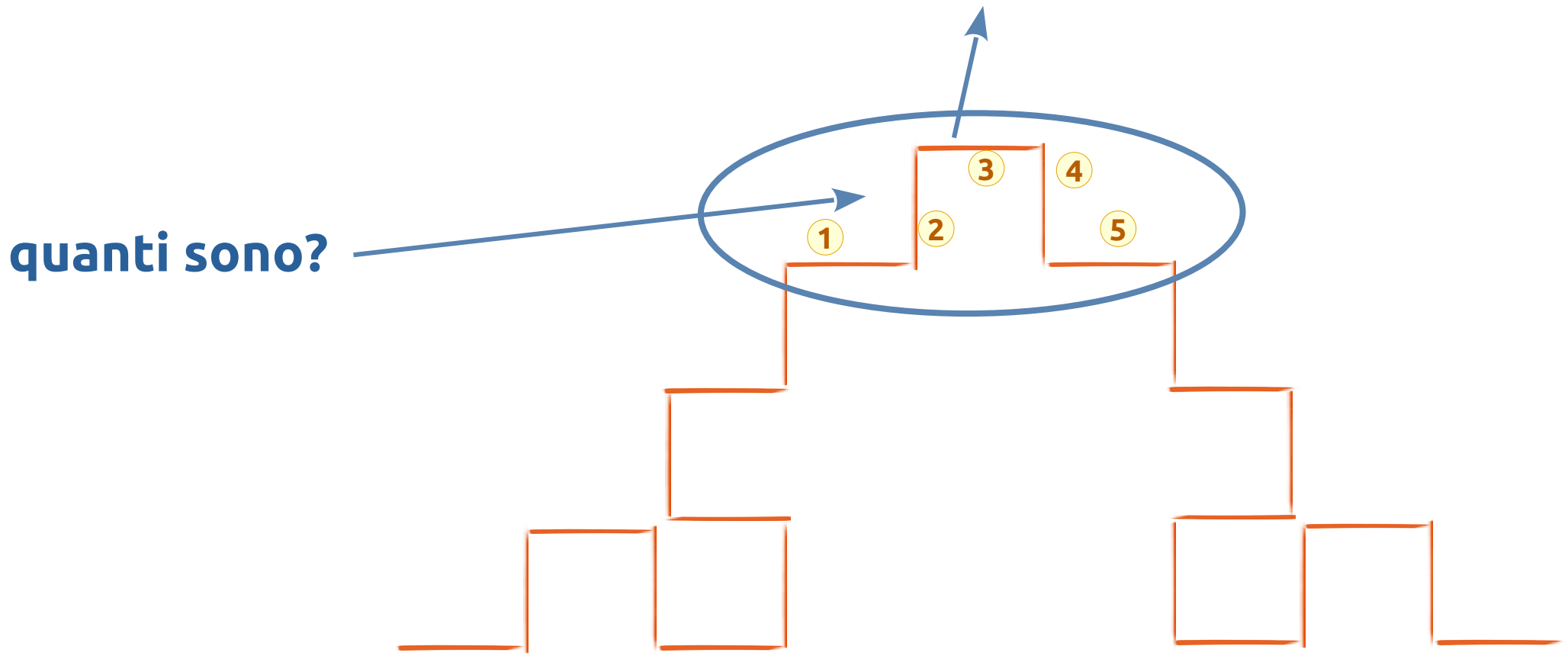


frattali
quanta strada percorre la tartaruga?

$$l = 1/3^2$$

5 per ogni “ripetizione”

quanti sono?

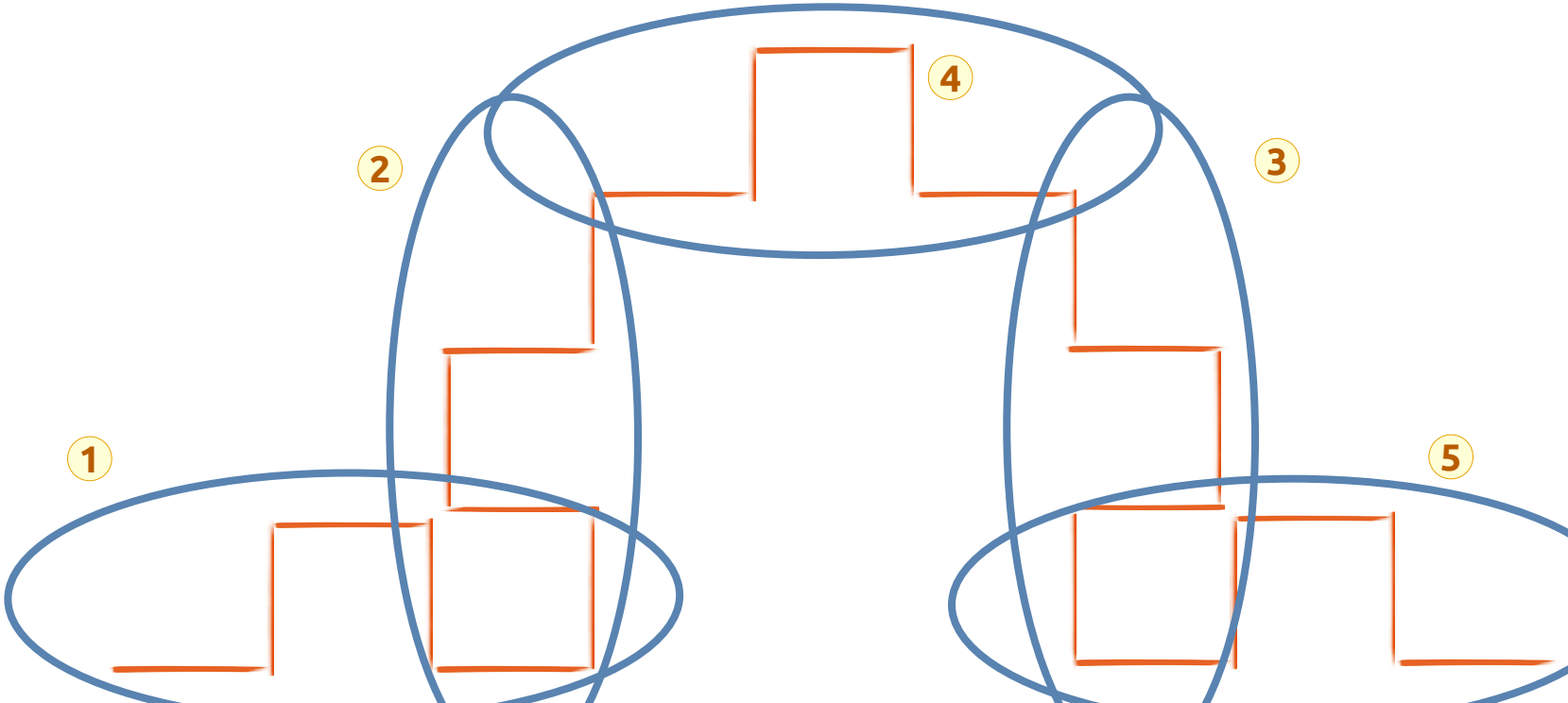


frattali
quanta strada percorre la tartaruga?

$$l = 1/3^2$$

5

quante ripetizioni?

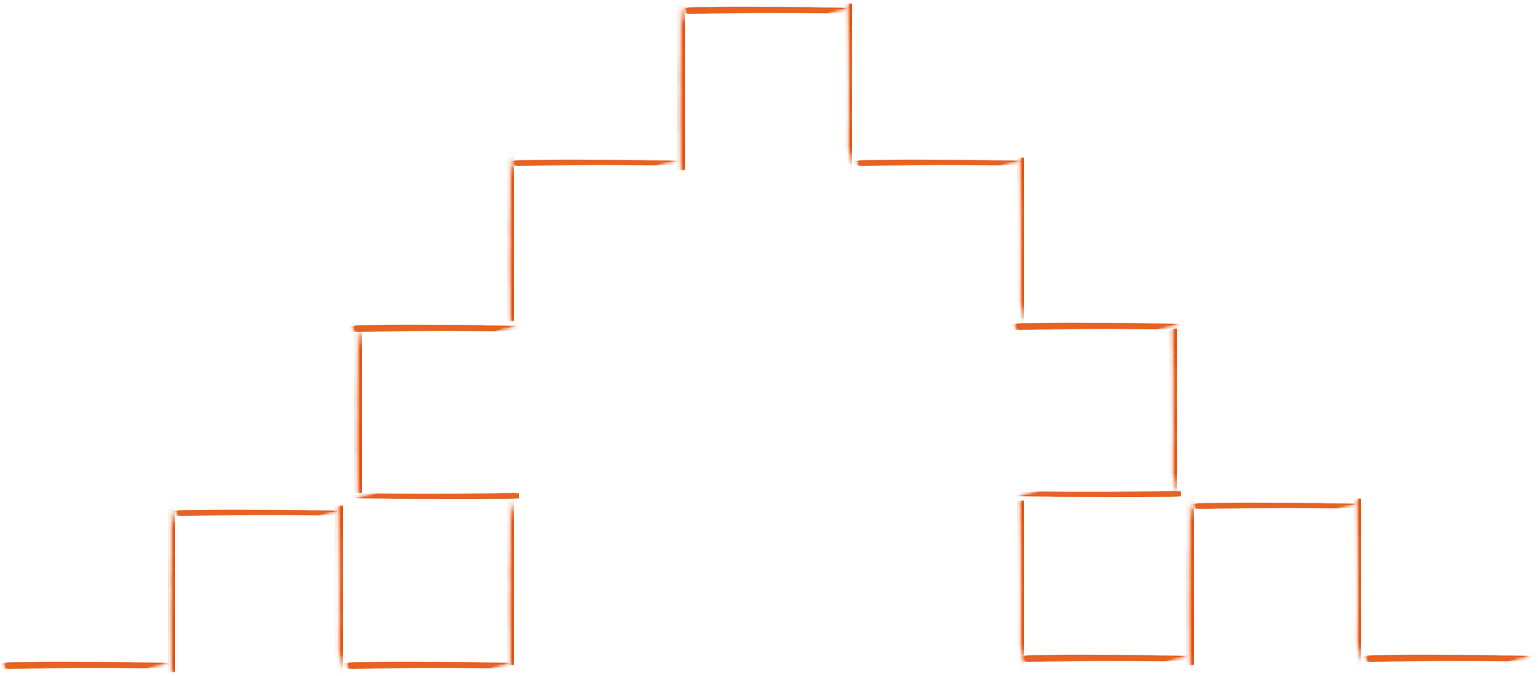


frattali
quanta strada percorre la tartaruga?

$l = 1/3^2$

5^2 segmenti

$l = 5^2/3^2$

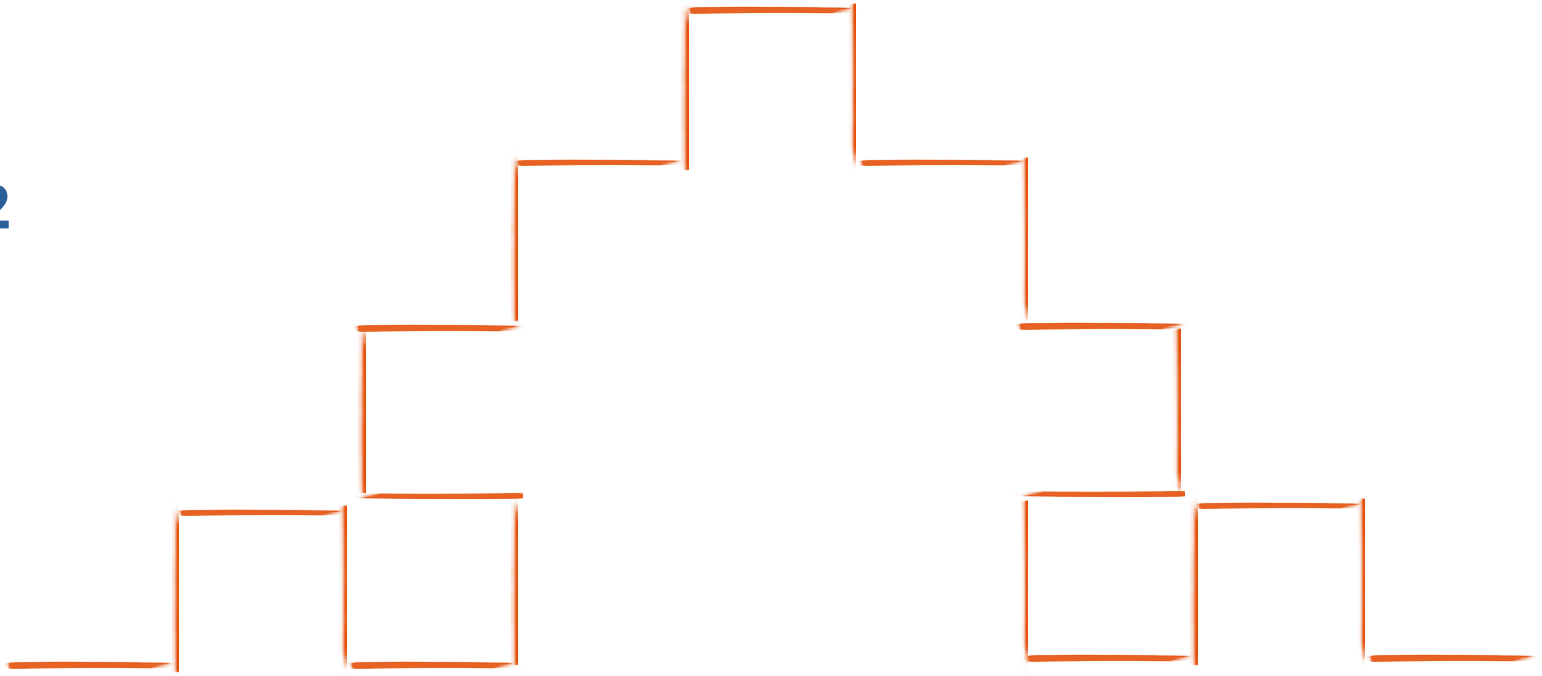


frattali
quanta strada percorre la tartaruga?

S → (1) A → AsAdAdAsA
(2) A → a

regole: **1, 1, 2**

$$l = 5^2/3^2$$



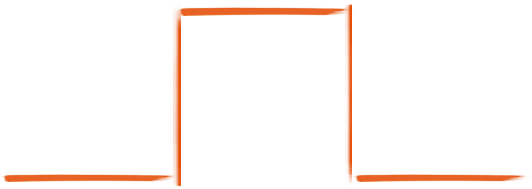
frattali
lunghezza

S → (1) A → AsAdAdAsA
(2) A → a

regole: **2**

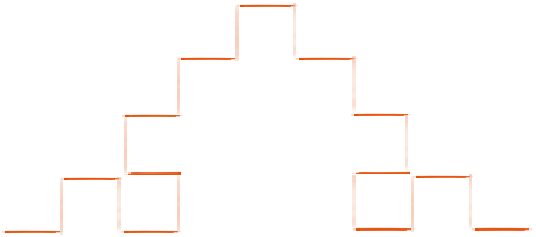
l = 1

regole: **1, 2**



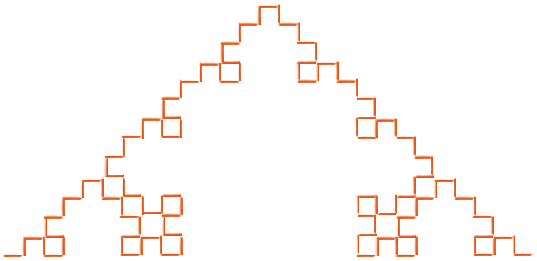
l = 5/3

regole: **1, 1, 2**



l = $5^2/3^2$

regole: **1, 1, 1, 2**



l = ?

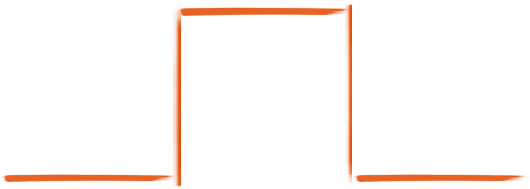
frattali
lunghezza

S → (1) A → AsAdAdAsA
(2) A → a

regole: **2**

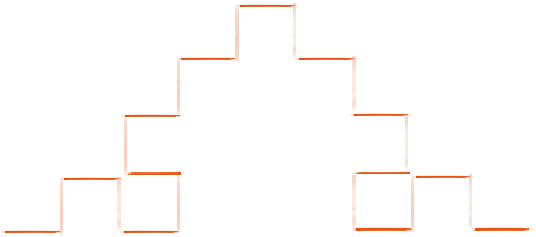
$l = (5/3)^0$

regole: **1, 2**



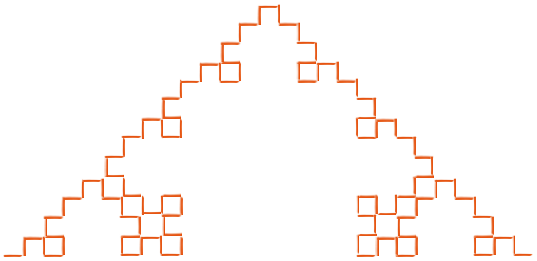
$l = (5/3)^1$

regole: **1, 1, 2**



$l = (5/3)^2$

regole: **1, 1, 1, 2**



$l = ?$

frattali
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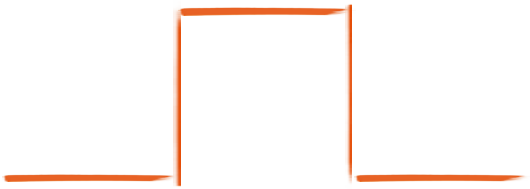
(2) A → a

regole: **2**

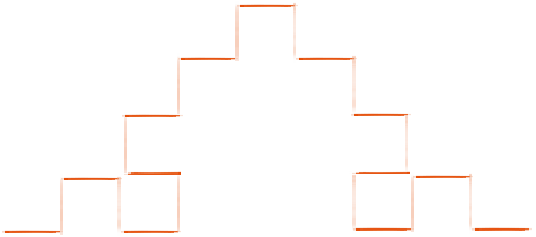
$l = (5/3)^0$

regole: **1, 2**



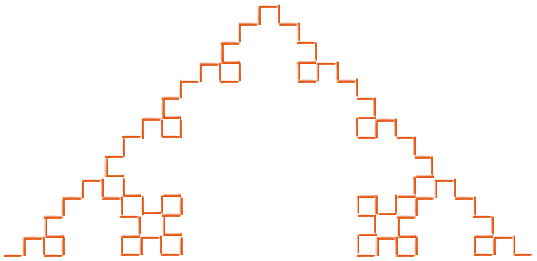
$l = (5/3)^1$

regole: **1, 1, 2**



$l = (5/3)^2$

regole: **1, 1, 1, 2**



$l = (5/3)^3$

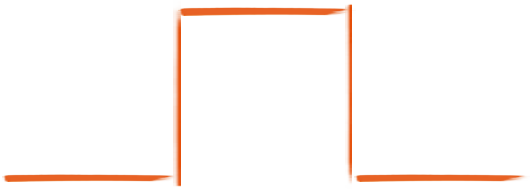
frattali
lunghezza

S → (1) A → AsAdAdAsA
(2) A → a

regole: **2**

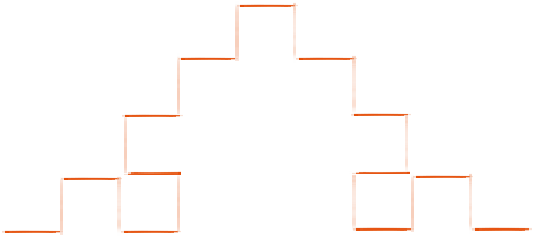
$l = (5/3)^0$

regole: **1, 2**



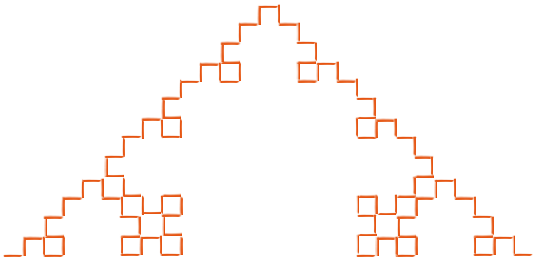
$l = (5/3)^1$

regole: **1, 1, 2**



$l = (5/3)^2$

regole: **1, 1, 1, 2**



$l = (5/3)^3$

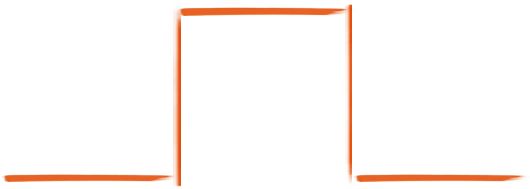
frattali
lunghezza

S → (1) A → AsAdAdAsA
(2) A → a

regole: **2**

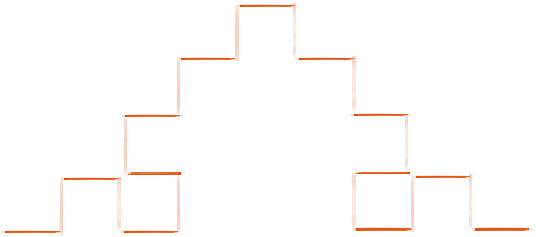
$l = (5/3)^0$

regole: **1, 2**



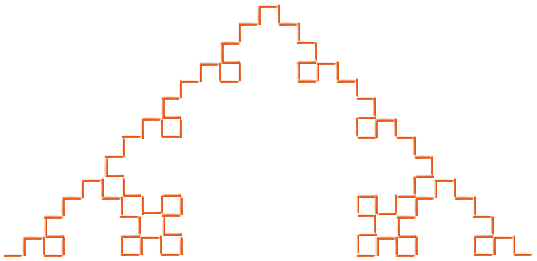
$l = (5/3)^1$

regole: **1, 1, 2**



$l = (5/3)^2$

regole: **1, 1, 1, 2**



$l = (5/3)^3$

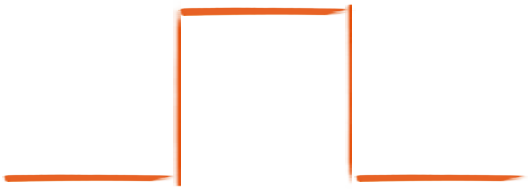
frattali
lunghezza

S → (1) A → AsAdAdAsA
(2) A → a

regole: 2

$l = (5/3)^0$

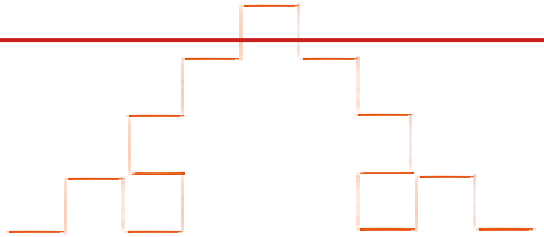
regole: 1, 2



$l = (5/3)^1$

1

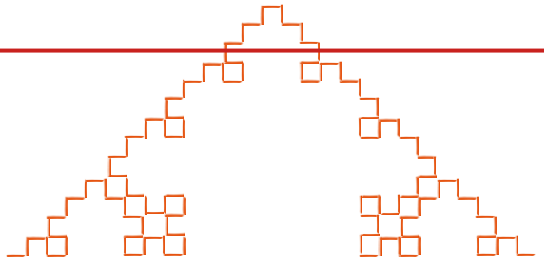
regole: 1, 1, 2



$l = (5/3)^2$

2

regole: 1, 1, 1, 2



$l = (5/3)^3$

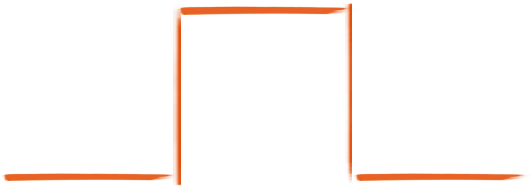
3

dimensione frattale
frattale “cappello”

S → **(1)** **A → AsAdAdAsA**

(2) **A → a**

$\Sigma = \{ a, s, d \}_{90°}$



5/3 → $\frac{\log(5)}{\log(3)}$

dimensione frattale

frattali
proprietà

1. autosimilarità

   il frattale è unione di copie di se stesso a scale differenti

2. struttura fine

   rivela dettagli ad ogni ingrandimento

3. dimensione non intera

   non 1, non 2, non 3, …

dimensione frattale
curva di von Koch

esercizio per casa!

S → **(1)** **A → AsAddAsA**

(2) **A → a**

$\Sigma = \{ a, s, d \}_{60°}$

? ⟶ $\dfrac{\log(?)}{\log(?)}$

dimensione frattale

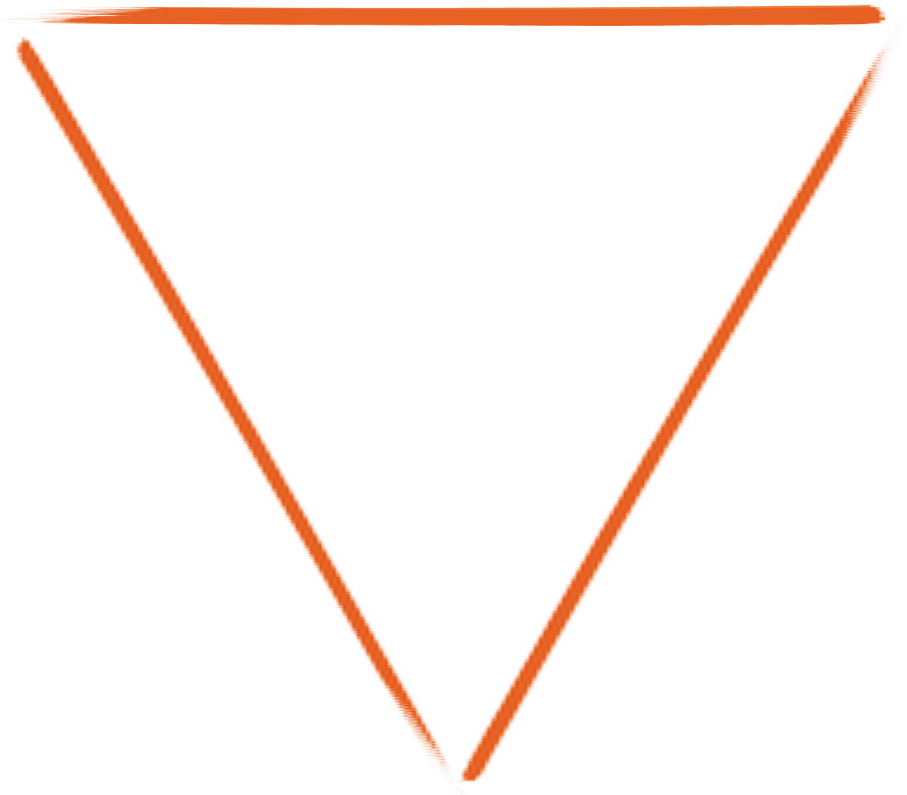
effetti speciali

linguaggi formali
il fiocco di neve

- grammatica

(1) S → A d d A d d A d d
(2) A → A s A d d A s A
(3) A → a

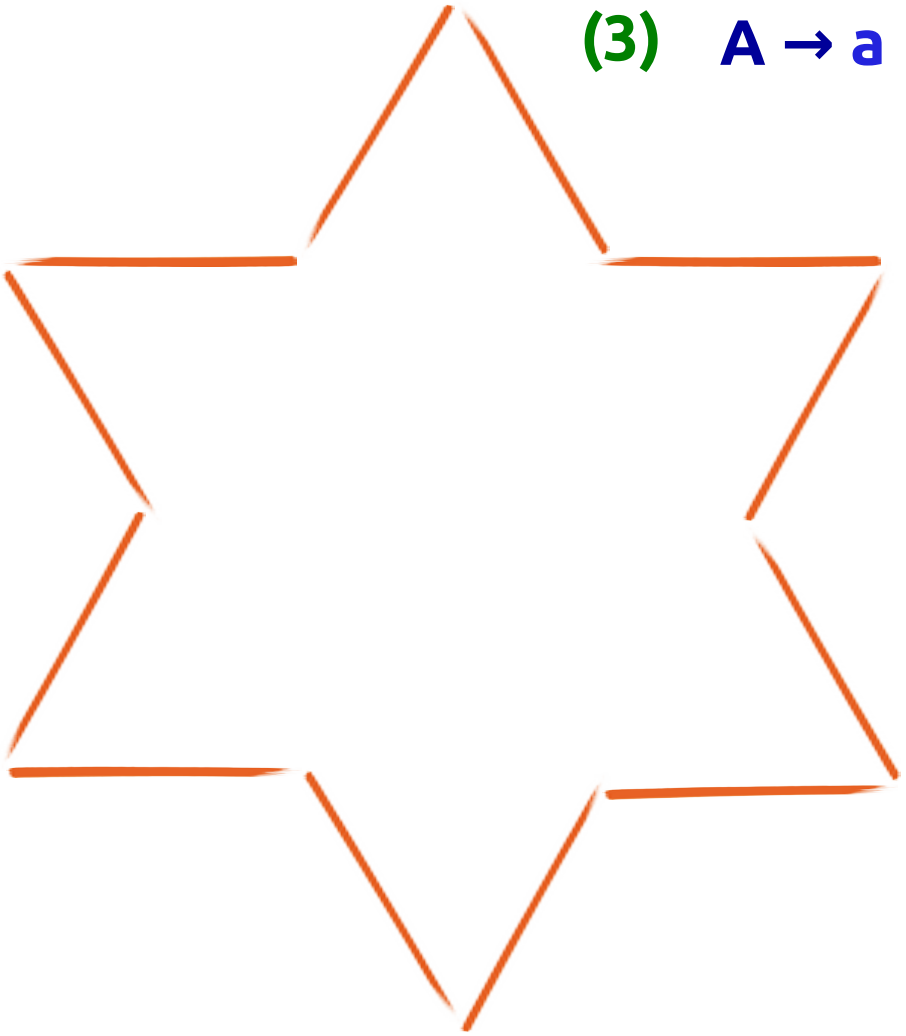
Regola 1



linguaggi formali
il fiocco di neve

- grammatica

(1) S → A d d A d d A d d
(2) A → A s A d d A s A
(3) A → a



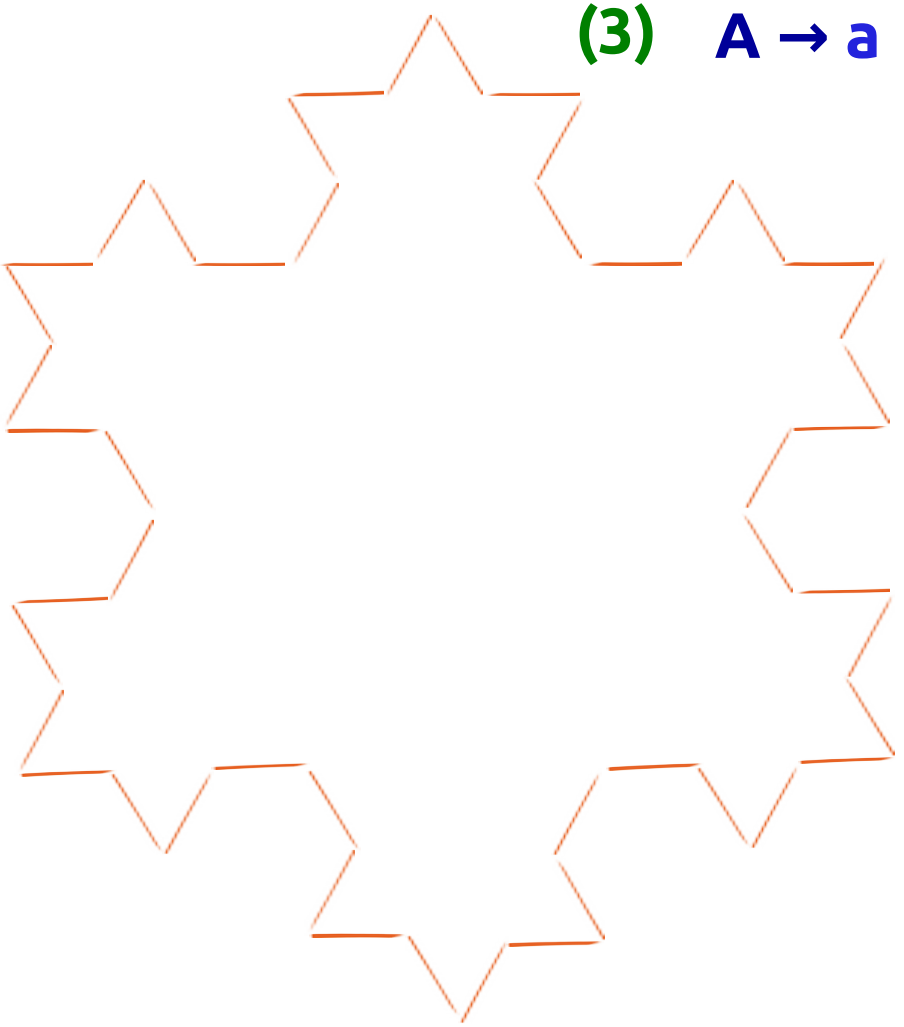
Regola 1

Regola 2

linguaggi formali
il fiocco di neve

- grammatica

(1) S → A d d A d d A d d
(2) A → A s A d d A s A
(3) A → a



Regola 1

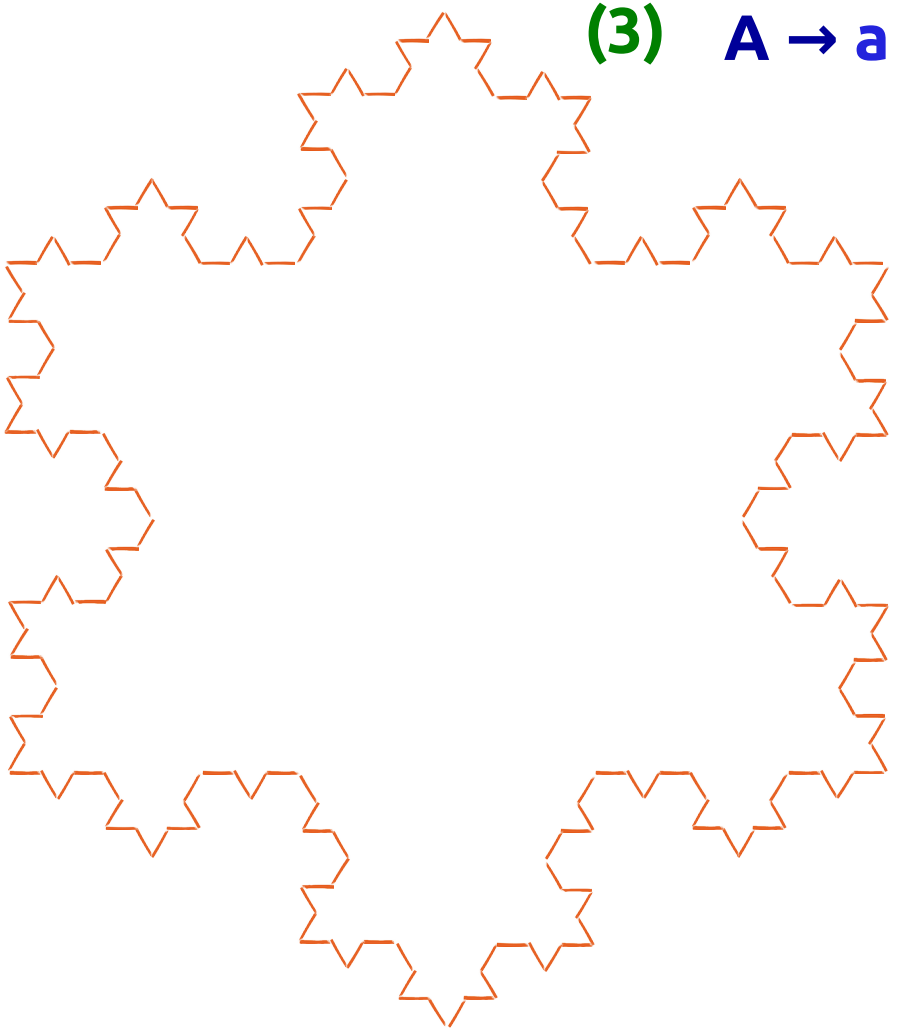
Regola 2

Regola 2

linguaggi formali
il fiocco di neve

- grammatica

(1) S → A d d A d d A d d
(2) A → A s A d d A s A
(3) A → a



Regola 1

Regola 2

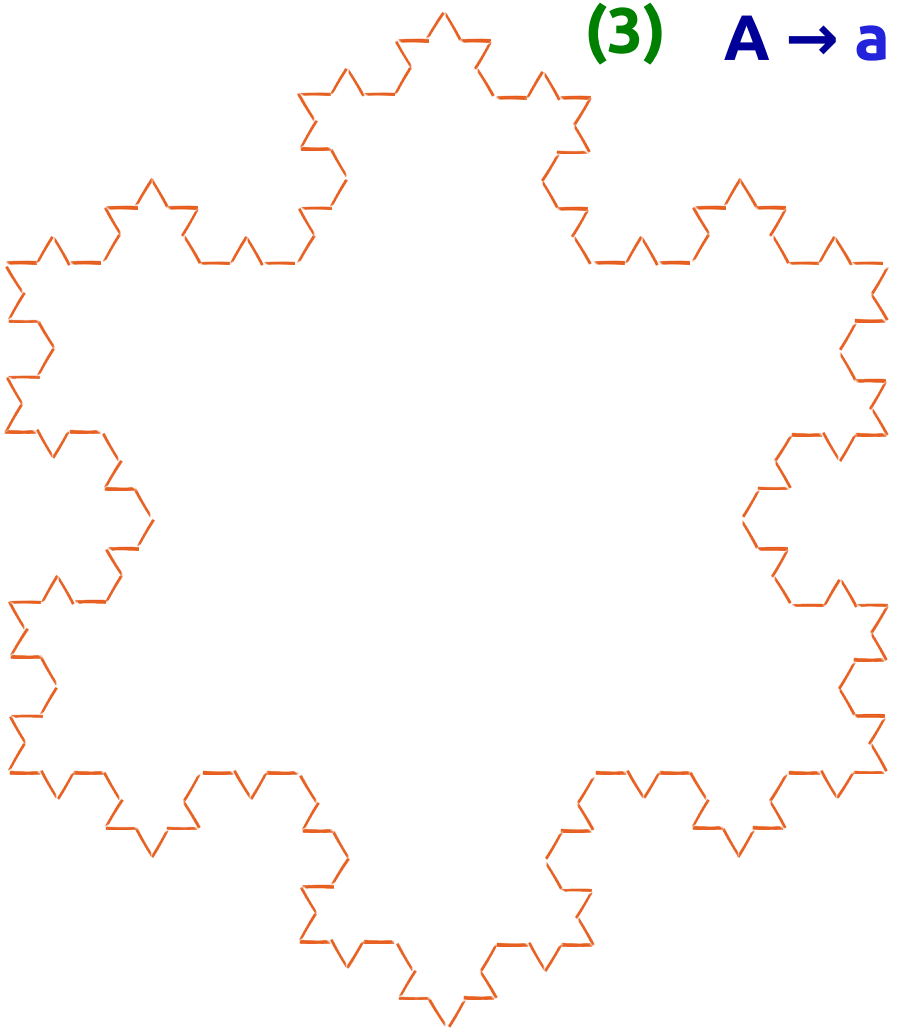
Regola 2

Regola 2

linguaggi formali
il fiocco di neve

- grammatica

(1) S → A d d A d d A d d
(2) A → A s A d d A s A
(3) A → a



Regola 1

Regola 2

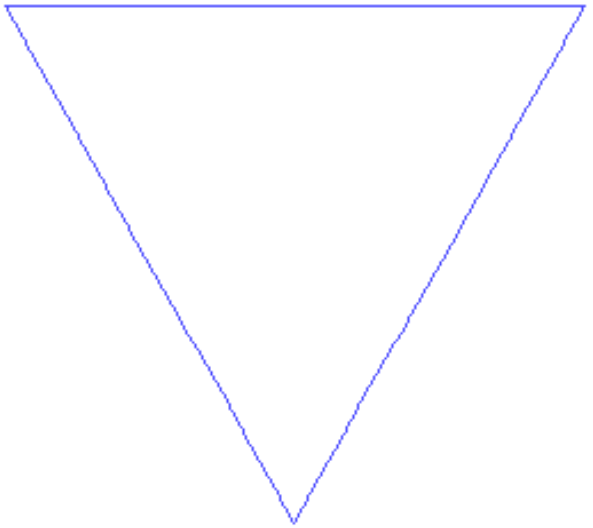
Regola 2

Regola 2

Regola 3

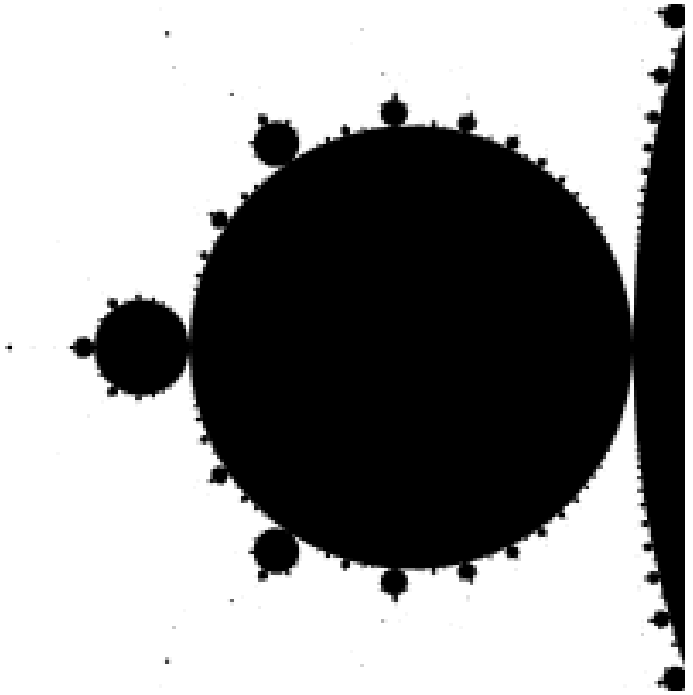
linguaggi formali
il fiocco di neve

- Tartaruga + autosimilarità:
  - Semplice algoritmo per costruire i **frattali**



linguaggi formali
fiocchi di neve, tappeti e broccoli

- Insieme di Mandelbrot



Grazie per la vostra pazienza!

Gaetano Impoco – gaetano @ impoco.it